

Stat 5101 (Geyer) Fall 2012  
Homework Assignment 11  
Due Wednesday, December 5, 2012

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

**11-1.** Suppose the random vector  $(X, Y)$  has variance matrix

$$\begin{pmatrix} 4 & -3 \\ -3 & 9 \end{pmatrix}$$

Find  $\text{sd}(X)$ ,  $\text{sd}(Y)$ , and  $\text{cor}(X, Y)$ . Note:  $\text{cor}(X, Y)$  is correlation not covariance.

**11-2.** Suppose  $\text{sd}(X) = 5$ ,  $\text{sd}(Y) = 7$ , and  $\text{cor}(X, Y) = 2/3$ . Find the variance matrix of the random vector  $(X, Y)$ .

**11-3.** Suppose  $X$  has the Poisson distribution with mean 100.

- (a) Calculate  $\Pr(X < 80)$  exactly.
- (b) Calculate  $\Pr(X < 80)$  using the normal approximation without correction for continuity.
- (c) Calculate  $\Pr(X < 80)$  using the normal approximation with correction for continuity.
- (d) Which of (b) and (c) is closer to correct?
- (e) Calculate  $\Pr(X > 120)$  exactly.
- (f) Calculate  $\Pr(X > 120)$  using the normal approximation without correction for continuity.
- (g) Calculate  $\Pr(X > 120)$  using the normal approximation with correction for continuity.
- (h) Which of (f) and (g) is closer to correct?

Be careful about weak and strict inequality.

**11-4.** Suppose  $X_1, \dots, X_{40}$  are IID random variables having the exponential distribution with rate parameter one. Let  $Y = X_1 + \dots + X_{40}$ .

- (a) Calculate  $\Pr(Y < 25)$  exactly.

- (b) Calculate  $\Pr(Y < 25)$  using the normal approximation.
- (c) Calculate  $\Pr(Y > 55)$  exactly.
- (d) Calculate  $\Pr(Y > 55)$  using the normal approximation.

**11-5.** Suppose  $X_1, \dots, X_{50}$  are IID random variables having mean 10 and standard deviation 5. Let  $\bar{X}_n = (X_1 + \dots + X_n)/n$ .

- (a) Calculate  $\Pr(\bar{X}_n < 9)$  using the normal approximation.
- (b) Calculate  $\Pr(\bar{X}_n > 11)$  using the normal approximation.

**11-6.** Suppose  $X_1, \dots, X_n$  are IID random variables having mean  $\mu$  and standard deviation  $\sigma > 0$ . Let  $\bar{X}_n = (X_1 + \dots + X_n)/n$ . Find a number  $c$ , which will be a function of  $\sigma$  and  $n$ , such that

$$\Pr(|\bar{X}_n - \mu| > c) \approx 0.05,$$

where the  $\approx$  means approximately equal using the normal approximation.

**11-7.** Suppose  $X_1, X_2, \dots$  is a sequence of IID random variables having mean  $\mu$  and standard deviation  $\sigma > 0$ . Let  $\bar{X}_n = (X_1 + \dots + X_n)/n$ . Does

$$\frac{n(\bar{X}_n - \mu)^2}{\sigma^2}$$

converge in distribution? If so, to what distribution does it converge?

Hint: CLT and continuous mapping theorem.

**11-8.** Suppose  $X_1, X_2, \dots$ , is a sequence of random variables,  $\theta$  is a constant, and

$$\sqrt{n}(X_n - \theta) \xrightarrow{\mathcal{D}} Y$$

where  $Y$  is any random variable. Show that this implies

$$X_n \xrightarrow{P} \theta.$$

Hint: Slutsky's theorem.

**11-9.** Suppose  $X_1, X_2, \dots$ , is a sequence of IID random variables, having mean  $\mu$  and standard deviation  $\sigma > 0$ . Suppose

$$S_n = g_n(X_1, \dots, X_n)$$

is some function of the data such that

$$S_n \xrightarrow{P} \sigma.$$

Show that

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{S_n} \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1).$$

Hint: CLT and Slutsky's theorem.