Stat 5101 Second Midterm Exam **Nov. 16, 2011**

Name ______________________________ Student ID ____________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 5 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability density function (PDF) given by

$$f_\theta(x) = \frac{\theta - 1}{x^\theta} \qquad 1 < x < \infty,$$

where $\theta > 4$ is a parameter.

(a) Find the mean of the distribution of X.

(b) Find the variance of the distribution of X.

2. [20 pts.] Suppose X is a random variable having probability density function (PDF) given by

$$f(x) = \frac{3(1 - x^2)}{4}, \qquad -1 < x < 1.$$

Find the PDF of the random variable $Y = \operatorname{asin}(X)$, where asin denotes the "arc sine" function whose inverse is the sine function. We choose the definition of the arc sine function that goes from $-\pi/2$ to $\pi/2$ as its argument goes from -1 to 1. The definition of a function describes the domain as well as the rule.

3. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{2\theta^2}{(\theta + x)^3}, \qquad 0 < x < \infty,$$

where $\theta > 0$ is a parameter. Find its distribution function (DF). Be sure to define the DF on the whole real line.

4. [20 pts.] Suppose the random vector (X, Y) has the PDF

$$f(x, y) = \frac{6(1 + x + y^2)}{11}, \qquad 0 < x < 1,\ 0 < y < 1.$$

(a) Find the conditional PDF of Y given X.

(b) Find the conditional PDF of X given Y.

5. [20 pts.] Suppose the conditional distribution of Y given X is $\text{Exp}(X)$, and suppose the marginal distribution of X is $\text{Gam}(\alpha, \lambda)$, where $\alpha > 0$ and $\lambda > 0$. What is the conditional distribution of X given Y? Since this is a brand name distribution, no integrals need be done, it is enough to name the distribution and give its parameters as a function of Y, α, and λ.