

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handout on “brand name distributions”. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability mass function (PMF) given by

x	-2	-1	0	1	2
$f(x)$	1/15	2/15	3/15	4/15	5/15

- (a) Calculate $E(X)$.

- (b) Calculate $\text{var}(X)$.

(c) Calculate $\Pr(X \neq 0)$.

2. [20 pts.] Suppose X is a random variable having PMF given by

x	1	2	3	4	5	6
$f(x)$	1/21	2/21	3/21	4/21	5/21	6/21

Find the PMF of the random variable $Y = g(X)$ where

$$g(x) = \begin{cases} x - 3 & x \geq 3 \\ 0, & x < 3 \end{cases}$$

3. [20 pts.] Suppose X_1 , X_2 , and X_3 are uncorrelated random variables all having mean μ and variance σ^2 . Find the mean vector and variance matrix of the random vector

$$\mathbf{Y} = \begin{pmatrix} X_1 \\ X_1 + X_2 \\ X_1 + X_2 + X_3 \end{pmatrix}$$

4. [20 pts.] Suppose the random vector (X, Y, Z) has PMF given by each of the following definitions. In each part say whether X , Y , and Z are independent random variables, and explain why or why not, as the case may be. In all parts of this question $S = \{-3, -2, -1, 0, 1, 2, 3\}$.

(a)

$$f(x, y, z) = \frac{1}{343}, \quad (x, y, z) \in S^3$$

(b)

$$f(x, y, z) = \frac{1}{84}, \quad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

(c)

$$f(x, y, z) = \frac{|x + y + z|}{960}, \quad (x, y, z) \in S^3$$

(d)

$$f(x, y, z) = \frac{|x + y + z|}{264}, \quad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

(e)

$$f(x, y, z) = \frac{|xyz|}{1728}, \quad (x, y, z) \in S^3$$

(f)

$$f(x, y, z) = \frac{|xyz|}{480}, \quad (x, y, z) \in S^3 \text{ and } x \leq y \leq z$$

5. [20 pts.] Suppose 4 balls that are indistinguishable except for color are placed in an urn, and suppose 2 are red and 2 are white.

(a) Suppose 3 balls are drawn from the urn *without replacement*. What is the probability mass function of the random variable X that is the number of red balls drawn?

Do not forget that specification of a function (the PMF here) includes saying what the domain is as well as a formula that transforms arguments to values. You may instead specify the PMF as a table (first row x , second row $f(x)$).

(b) Suppose 3 balls are drawn from the urn *with replacement*. What is the probability mass function of the random variable X that is the number of red balls drawn?