Stat 5101 First Midterm Exam **October 14, 2009**

Name ______________________________ Student ID __________________

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handout on "brand name distributions". Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability mass function (PMF) given by

x	1	2	3	4	5
$f(x)$	1/7	2/7	1/7	2/7	1/7

(a) Calculate $E(X)$.

(b) Calculate $\operatorname{var}(X)$.

(c) Calculate $\Pr(X > 3)$.

2. [20 pts.] Suppose X is a random variable having PMF given by

x	1	2	3	4	5	6	7
$f(x)$	$1/28$	$1/14$	$3/28$	$1/7$	$5/28$	$3/14$	$1/4$

Find the PMF of the random variable $Y = (X - 2)(X - 4)(X - 6)$.

3. [20 pts.] Suppose $\mathbf{X}$ is a random vector having mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

And suppose

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

and define the random vector $\mathbf{Y} = \mathbf{B}\mathbf{X}$ (the right hand side being a matrix multiplication).

(a) Calculate the mean vector of $\mathbf{Y}$.

(b) Calculate the variance matrix of $\mathbf{Y}$.

4. [20 pts.] Suppose the random vector (X, Y) has PMF given by each of the following definitions. In each part say whether X and Y are independent random variables, and explain why or why not, as the case may be. (Recall that $\mathbb{N}$ denotes the set of natural numbers 0, 1, ...)

(a)

$$f(x, y) = \frac{x + 2y}{225}, \qquad (x, y) \in \mathbb{N}^2 \text{ and } 1 \leq x \leq 5 \text{ and } 1 \leq y \leq 5$$

(b)

$$f(x, y) = \frac{xy^2}{825}, \qquad (x, y) \in \mathbb{N}^2 \text{ and } 1 \leq x \leq 5 \text{ and } 1 \leq y \leq 5$$

(c)

$$f(x,y) = \frac{x+2y}{145}, \qquad (x,y) \in \mathbb{N}^2 \text{ and } 1 \le x \le y \le 5$$

(d)

$$f(x,y) = \frac{xy^2}{602}, \qquad (x,y) \in \mathbb{N}^2 \text{ and } 1 \le x \le y \le 5$$

5. [20 pts.] Suppose 8 balls that are indistinguishable except for color are placed in an urn, and suppose 5 are red and 3 are white.

   (a) Suppose 4 balls are drawn from the urn *without replacement.* What is the probability that 2 balls are red and 2 balls are white?

   (b) Suppose 4 balls are drawn from the urn *with replacement.* What is the probability that 2 balls are red and 2 balls are white?