

Stat 5101 (Geyer) Fall 2009  
Homework Assignment 3  
Due Wednesday, September 30, 2009

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

**3-1.** Suppose that  $p$  is a PMF on a sample space  $\Omega$ , suppose  $X$  and  $Y$  are random variables in this probability model. Prove the following statements.

- (a)  $E(X + Y) = E(X) + E(Y)$ .
- (b) If  $X(\omega) \geq 0$  for all  $\omega \in \Omega$ , then  $E(X) \geq 0$ .
- (c) If  $Y(\omega) = a$  for all  $\omega \in \Omega$ , then  $E(XY) = aE(X)$ .
- (d) If  $Y(\omega) = 1$  for all  $\omega \in \Omega$ , then  $E(Y) = 1$ .

Do not use the axioms (these are the axioms). The problem is to prove that these statements follow from our earlier definition of PMF and expectation.

**3-2.** Suppose  $X$  has the uniform distribution on the set  $\{1, 2, 3, 4\}$ , and suppose  $Y = X^2$ .

- (a) Calculate  $E(X)$ .
- (b) Calculate  $E(Y)$ .
- (c) Calculate  $E(Y/X)$ .
- (d) Calculate  $E(Y)/E(X)$ .
- (e) Compare your answers in (c) and (d). Are they the same? Should they be the same?

**3-3.** Suppose  $E(X) = 3$  and  $E(Y) = 4$ . Calculate  $E(5X + Y)$ .

**3-4.** Suppose  $X$  is a random variable having PMF given by

$x$	1	2	3	4	5
$f(x)$	1/9	2/9	3/9	2/9	1/9

- (a) Calculate  $E(X)$ .
- (b) Calculate  $\text{var}(X)$ .

**3-5.** Suppose  $X$  is a  $\text{Ber}(p)$  random variable and  $Y = 2X - 1$ .

- (a) Calculate  $E(Y)$ .
- (b) Calculate  $\text{var}(Y)$ .
- (c) Calculate  $E(Y^2)$ .
- (d) Calculate  $\text{var}(Y^2)$ .

**3-6.** Suppose  $X$  has the discrete uniform distribution on the set

$$\{x \in \mathbb{Z} : l \leq x \leq u\}$$

where  $l$  and  $u$  are integers with  $l < u$ .

- (a) Calculate  $E(X)$ .
- (b) Calculate  $\text{var}(X)$ .

**3-7.** Suppose  $X$  and  $Y$  are random variables in the same probability model, and suppose  $a$ ,  $b$ ,  $c$ , and  $d$  are constants. Prove that

$$\text{cov}(a + bX, c + dY) = bd \text{cov}(X, Y)$$

**3-8.** Suppose

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

- (a) Calculate  $\mathbf{AB}$ .
- (b) Calculate  $\mathbf{BA}$ .
- (c) Compare your answers in (a) and (b). Are they the same? Should they be the same?

**3-9.** Suppose  $X$  is a random variable with mean  $\mu$  and variance  $\sigma^2$ . Calculate the mean vector and variance matrix of the random vector  $\mathbf{Y} = (X, 2 + 3X)$ .

**3-10.** Suppose  $\mathbf{X}$  is a random variable with mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} v_1 & c \\ c & v_2 \end{pmatrix}$$

Calculate the mean vector and variance matrix of the random vector  $\mathbf{Y} = \mathbf{a} + \mathbf{B}\mathbf{X}$  where

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

## Review Problems from Previous Midterm

**3-11.** Suppose  $X$  is a random variable having probability mass function (PMF) given by

$x$	-2	-1	0	1	2
$f(x)$	1/3	1/6	0	1/6	1/3

(a) Calculate  $E(X)$ .

(b) Calculate  $\text{var}(X)$ .

**3-12.** Suppose  $X$  is a random variable having PMF given by

$x$	-2	-1	0	1	2
$f(x)$	1/9	2/9	1/3	2/9	1/9

(a) Find the PMF of the random variable  $Y = X^2$ .

(b) Calculate  $\Pr(Y > 0)$ .

**3-13.** Suppose the random vector  $(X, Y)$  has PMF given by

$$f(x, y) = \frac{x^2 y}{90}, \quad x = -2, -1, 0, 1, 2, \quad y = 2, 3, 4.$$

Are  $X$  and  $Y$  independent random variables? Explain why or why not, as the case may be.