Stat 5101 Second Midterm Exam   **November 12, 2008**

Name ————————————————— Student ID ———————————

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on "brand name distributions" and Greek letters. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 5 pages and 6 problems.

1. [20 pts.] Suppose X is a random variable having probability density function (PDF) given by

$$f(x) = \frac{6}{5}(x + x^2), \qquad 0 < x < 1.$$

   (a) Calculate $E(X)$.

   (b) Calculate $\text{var}(X)$.

2. [20 pts.] Suppose X is a random variable having PDF given by

$$f(x) = \frac{2}{x^3}, \qquad 1 < x < \infty.$$

Find the PDF of the random variable $Y = \log(X)$.

3. [20 pts.]

   (a) For what real numbers α does there exist a constant $c(\alpha)$ such that the function
$$f_\alpha(x) = \frac{c(\alpha)}{x^\alpha}, \qquad x > 1$$
   is a PDF?

   (b) Suppose α is such that f_α is a PDF. If X is a random variable having this PDF, for what positive real numbers β does $E(X^\beta)$ exist?

4. [10 pts.] Calculate the PDF corresponding to the DF

$$F(x) = \begin{cases} 0, & x \leq 0 \\ (x + x^2)/2, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

5. [10 pts.] Calculate the DF corresponding to the PDF

$$f(x) = \frac{\cos(x)}{2}, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Define the DF on the whole real line.

6. [20 pts.] Suppose the random variable X has the PDF (not the DF) given by

$$f(x) = 2x, \qquad 0 < x < 1.$$

Find the median (not the mean) of the distribution of X.