

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handout on “brand name distributions”. Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for numbers with no indication of where they came from.

The points for the questions total to 100. There are 4 pages and 5 problems.

1. [20 pts.] Suppose X is a random variable having probability mass function (PMF) given by

x	-2	-1	0	1	2
$f(x)$	1/3	1/6	0	1/6	1/3

- (a) Calculate $E(X)$.

- (b) Calculate $\text{var}(X)$.

2. [20 pts.] Suppose X is a random variable having PMF given by

x	-2	-1	0	1	2
$f(x)$	1/9	2/9	1/3	2/9	1/9

(a) Find the PMF of the random variable $Y = X^2$.

(b) Calculate $\Pr(Y > 0)$.

3. [20 pts.] Suppose \mathbf{X} is a random vector having mean vector

$$\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

and variance matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

(a) Calculate $E(X_1 + 2X_2 + 3X_3)$.

(b) Calculate $\text{var}(X_1 + 2X_2 + 3X_3)$.

4. [20 pts.] Suppose a basketball player is shooting free throws and the results (success or failure) are considered to be independent and identically distributed Bernoulli random variables with success probability $3/4$ (code success as one and failure as zero). The player shoots three free throws. What is the probability that she makes at least one?

5. [20 pts.] Suppose the random vector (X, Y) has PMF given by

$$f(x, y) = \frac{x^2 y}{90}, \quad x = -2, -1, 0, 1, 2, \quad y = 2, 3, 4.$$

Are X and Y independent random variables? Explain why or why not, as the case may be.