# Stat 5101 (Geyer) Fall 2008
# Homework Assignment 9
# Due Wednesday, November 19, 2008

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

**9-1.** Suppose the random vector $(X, Y)$ has the PDF

$$f(x, y) = \frac{1}{3}(x + y + xy)e^{-x-y}, \qquad 0 < x < \infty,\ 0 < y < \infty. \tag{1}$$

Find the marginal PDF of $X$.

**9-2.** Suppose the random vector $(X, Y)$ has the uniform distribution on the triangle

$$\{\, (x, y) \in \mathbb{R}^2 : 0 < x < y < 1 \,\} \tag{2}$$

(a) Find the marginal PDF for $X$.

(b) Find the marginal PDF for $Y$.

Hint: be careful about ranges of integration; if $y$ is fixed, then the range of $x$ is $0 < x < y$; if $x$ is fixed, then the range of $y$ is $x < y < 1$.

**9-3.** Suppose the conditional distribution of $Y$ given $X$ has PDF

$$f(y \mid x) = \frac{6(x + xy + y^2)}{2 + 9x}, \qquad 0 < y < 1.$$

(a) Find $E(Y \mid x)$.

(b) Find $\operatorname{var}(Y \mid x)$.

**9-4.** Suppose the conditional distribution of $Y$ given $X$ has is $\mathcal{N}(X, X)$.

(a) Find $E(Y \mid x)$.

(b) Find $\operatorname{var}(Y \mid x)$.

(c) Find $E(Y^2 \mid x)$.

**9-5.** Suppose the random vector $(X, Y)$ has the PDF (1) Find the conditional PDF of $Y$ given $X$.

**9-6.** Suppose the random vector $(X, Y)$ has the uniform distribution on the triangle (2)

(a) Find the conditional PDF of $Y$ given $X$.

(b) Find the conditional PDF of $X$ given $Y$.

(c) Find the conditional mean of $Y$ given $X$.

(d) Find the conditional mean of $X$ given $Y$.

(e) Find the conditional median of $Y$ given $X$.

(f) Find the conditional median of $X$ given $Y$.

**9-7.** Suppose the random vector $(X, Y)$ has the PDF

$$f(x, y) = \frac{4}{5}(x + y + xy), \qquad 0 < x < 1, \ 0 < y < 1.$$

(a) Find the conditional PDF of $Y$ given $X$.

(b) Find the conditional mean of $Y$ given $X$.

(c) Find the conditional median of $Y$ given $X$.

**9-8.** Suppose the conditional distribution of $Y$ given $X$ is $\text{Bin}(n, X)$, where $n$ is a known positive integer, and suppose the marginal distribution of $X$ is $\text{Beta}(\alpha_1, \alpha_2)$. What is the conditional distribution of $X$ given $Y$? Since this is a brand name distribution, no integrals need be done, it is enough to name the distribution and give its parameters as a function of $Y$ and $n$.

**9-9.** Suppose the conditional distribution of $Y$ given $X$ is $\mathcal{N}(\mu, 1/X)$, where $\mu$ is a known real number, and suppose the marginal distribution of $X$ is $\text{Gam}(\alpha, \lambda)$. What is the conditional distribution of $X$ given $Y$? Since this is a brand name distribution, no integrals need be done, it is enough to name the distribution and give its parameters as a function of $Y$ and $n$.

**9-10.** Suppose

$$\begin{aligned} E(Y \mid X) &= X \\ \text{var}(Y \mid X) &= 3X^2 \end{aligned}$$

and suppose the marginal distribution of $X$ is $\mathcal{N}(\mu, \sigma^2)$.

(a) Find $E(Y)$.

(b) Find $\text{var}(Y)$.

**9-11.** Suppose $X_1, \ldots, X_N$ are IID having mean $\mu$ and variance $\sigma^2$ where $N$ is a Poi($\lambda$) random variable independent of all of the $X_i$. Let

$$Y = \sum_{i=1}^{N} X_i,$$

with the convention that $N = 0$ implies $Y = 0$.

(a) Find $E(Y)$.

(b) Find $\operatorname{var}(Y)$.

**9-12.** Suppose that the conditional distribution of $Y$ given $X$ is Poi($X$), and suppose that the marginal distribution of $X$ is Gam($\alpha, \lambda$). Show that the marginal distribution of $Y$ is a negative binomial distribution in the extended sense discussed in the brand name distributions handout in which the shape parameter need not be an integer. Identify the parameters of this negative binomial distribution (which are functions of $\alpha$ and $\lambda$).

**9-13.** Suppose that $X$ and $Y$ are independent Poisson random variables having means $\mu_1$ and $\mu_2$, respectively. Show that the conditional distribution of $X$ given $X + Y$ is binomial. Identify the parameters of this binomial distribution (which are functions of $\mu_1$ and $\mu_2$ and $X + Y$).

**9-14.** Suppose that $X$ has the Geo($p$) distribution, show that the conditional distribution of $X - k$ given $X \geq k$ also has the Geo($p$) distribution.

**9-15.** Suppose service times of customers in line at a bank teller are IID Exp($\lambda$) random variables. Suppose when you arrive there are nine customers in line in front of you (ten customers including you). What is the mean and standard deviation of the time until you are served (including your service time)?

**9-16.** Suppose cars crossing the midpoint of the new 35W bridge about 8:00 am on a weekday can be modeled as a Poisson process with rate 20 cars per minute. What is the mean and standard deviation of the number of cars crossing in 5 minutes?