Stat 8112 (Geyer) Spring 2013
Homework Assignment 7
Due Monday April 29, 2013

7-1. Ferguson problem 21-1.

7-2. Show that a gamma mixture of Poisson is negative binomial, that is, suppose the conditional distribution of X given μ is Poisson(μ) and the marginal distribution of μ is Gamma(α, λ) and show that the marginal distribution of X is negative binomial with a possibly noninteger sample size (if you are not familiar with this notion, see the Stat 5102 handout on "brand-name distributions" at `http://www.stat.umn.edu/geyer/5102/notes/brand.pdf`).

7-3. Suppose we assume X_1, X_2, ... are IID Poisson(μ) so the MLE for μ is $\overline{X}_n$, but the model is misspecified and the data actually have the negative binomial distribution with possibly noninteger sample size derived in the preceding problem. Derive the asymptotic distribution of the MLE (under this misspecification) two different ways. First, use the CLT and the properties of the negative binomial distribution. Second, use the formula for inverse Godambe information and the characterization of μ^* as the maximizer of Kullback-Leibler information.

7-4. An AR(1) (autoregressive of order one) time series is defined as follows. Let Z_1, Z_2, ... be IID standard normal. Let ρ and σ be unknown parameters with $\sigma > 0$. Let X_1 have any distribution and inductively define

$$X_{n+1} = \rho X_n + \sigma Z_n, \qquad n = 1, 2, \ldots.$$

Then we say X_1, X_2, ... is an AR(1) time series centered at zero. If μ is another unknown parameter, then we say Y_1, Y_2, ... is a general AR(1) time series, where

$$Y_n = \mu + X_n, \qquad n = 1, 2, \ldots.$$

(a) Show that if X_1 has a certain normal distribution (with parameters you are to figure out) that the AR(1) time series centered at zero is a strictly stationary stochastic process when ρ has certain values (which you are also to figure out).

(b) Show that if ρ is not in the set of values you determined in part (a) that there does not exist any distribution for X_1 that makes the process strictly stationary. Hint: derive the relationship between the characteristic functions of X_1 and X_2 (which must be the same under stationarity) and show that these cannot be made equal for certain values of ρ. Use that fact that for any characteristic function φ we have $|\varphi(t)| \leq 1$ for all t, where $|\cdot|$ is the complex modulus.

(c) Suppose we have a strictly stationary AR(1) time series with unknown parameters μ, ρ, and σ and want to estimate these using a composite likelihood approach with one term for the conditional distribution of Y_n given Y_{n-1} for $n = 2, \ldots, n$. Derive closed-form expressions for the conditional composite likelihood estimators of these three parameters.

(d) Derive the asymptotic normal distribution of these estimators. (Assume the regularity conditions for composite likelihood estimators hold.)

(e) Write down the log likelihood for this model. How different is it from the conditional composite log likelihood? Are there closed-form expressions for the MLE's. If not, why not?

(f) Suppose we have a strictly stationary AR(1) time series with unknown parameters μ, ρ, and σ and want to estimate these using a marginal composite likelihood approach with one term for the marginal distribution of the pair (Y_n, Y_{n-1}) for $n = 2, \ldots, n$. Derive closed-form expressions for the marginal composite likelihood estimators of these three parameters.