Stat 8112 (Geyer) Spring 2013
Homework Assignment 3
Due Friday February 22, 2013

3-1. Ferguson, Problem 6-1.

3-2. Ferguson, Problem 6-3.

3-3. Ferguson, Problem 7-1.

3-4. Ferguson, Problem 7-2.

3-5. Ferguson, Problem 7-3

3-6. Ferguson, Problem 7-4

3-7. Ferguson, Problem 7-6

3-8. Suppose X_1, X_2, $\ldots$ are IID $\text{Unif}(0, \theta)$. Define

$$X_{(n)} \max_{1 \le i \le n} X_i.$$

(a) Show that

$$X_{(n)} \xrightarrow{\text{a. s.}} \theta, \qquad \text{as } n \to \infty.$$

(b) Show that

$$n\bigl(\theta - X_{(n)}\bigr) \xrightarrow{\mathcal{L}} \text{Exp}(1/\theta), \qquad \text{as } n \to \infty.$$

3-9. Suppose X_1, X_2, $\ldots$ are IID. Define

$$\alpha_k = E(X_i^k)$$
$$\hat{\alpha}_{k,n} = \frac{1}{n} \sum_{i=1}^{n} X_i^k$$

(a) Show that

$$\hat{\alpha}_{k,n} \xrightarrow{\text{a. s.}} \alpha_k, \qquad \text{as } n \to \infty$$

provided k-th moments exist.

(b) Assume $(2k)$-th moments exist, and find the multivariate asymptotic normal distribution of the sequence of random vectors $(\hat{\alpha}_{1,n}, \ldots, \hat{\alpha}_{k,n})$.

3-10. Suppose X_1, X_2, ... are IID. Define

$$\begin{aligned}
\mu &= E(X_i) \\
\mu_k &= E\{(X_i - \mu)^k\} \\
\overline{X}_n &= \frac{1}{n} \sum_{i=1}^{n} X_i \\
\hat{\mu}_{k,n} &= \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X}_n)^k
\end{aligned}$$

(a) Show that

$$\hat{\mu}_{k,n} \xrightarrow{\text{a. s.}} \mu_k, \qquad \text{as } n \to \infty$$

provided k-th moments exist.

(b) Assume $(2k)$-th moments exist, and find the asymptotic normal distribution of the sequence of random vectors $(\overline{X}_n, \hat{\mu}_{2,n}, \ldots, \hat{\mu}_{k,n})$.

Hint: first find the asymptotic normal distribution of $(\overline{X}_n, \mu^*_{2,n}, \ldots, \mu^*_{k,n})$, where

$$\mu^*_{k,n} = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^k.$$

Then use the binomial theorem to write the $\hat{\mu}_{k,n}$ in terms of $\overline{X}_n$ and the $\mu^*_{k,n}$.

3-11. Suppose X_1, X_2, ... are IID $\text{Gam}(\alpha, \lambda)$, where λ is the rate parameter. Define

$$\begin{aligned}
\hat{\alpha}_n &= \frac{\overline{X}_n^2}{V_n} \\
\hat{\lambda}_n &= \frac{\overline{X}_n}{V_n}
\end{aligned}$$

where

$$\begin{aligned}
\overline{X}_n &= \frac{1}{n} \sum_{i=1}^{n} X_i \\
V_n &= \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2
\end{aligned}$$

Find the bivariate asymptotic normal distribution of the sequence of random vectors $(\hat{\alpha}_n, \hat{\lambda}_n)$.