Stat 8112 (Geyer) Spring 2013
Homework Assignment 1
Due Friday, February 1, 2013

1-1. Suppose X_n and X are integer-valued random variables, and let

$$f_n(x) = \Pr(X_n = x)$$
$$f(x) = \Pr(X = x)$$

Show that $X_n \xrightarrow{\mathcal{L}} X$ if and only if $f_n(x) \to f(x)$ for all integers x.

Can you generalize the condition? Find a weaker description of the setup (the definition of X_n, X, f_n, and f) so that the conclusion (the "show that") still holds.

1-2. Suppose X_n is Binomial(m_n, p_n) and X is Poisson(μ). Show that $m_n \to \infty$ and $m_n p_n \to \mu$ implies $X_n \xrightarrow{\mathcal{L}} X$ without using characteristic functions or moment generating functions.

1-3. Suppose X_n is Geometric(p_n) and X is Exponential(λ), and suppose ε_n is a decreasing sequence of real numbers converging to zero. Find conditions that imply $\varepsilon_n X_n \xrightarrow{\mathcal{L}} X$.

The geometric and exponential distributions have a variety of more or less standard parameterizations. To make things easier for the grader, everyone use the parametizations that R uses (X_n is the number of *failures* (not trials) before the first success in an independent and identically distributed (IID) sequence of Bernoulli trials with success probability p_n, and X has probability density function $\lambda e^{-\lambda x}$).

1-4. Give an example where $X_n \xrightarrow{\mathcal{L}} \mathcal{N}(0, 1)$ but $\operatorname{var}(X_n) = \infty$ for all n.

1-5. Ferguson, Problem 1-6.

1-6. Ferguson, Problem 2-6.

1-7. Ferguson, Problem 2-7.

1-8. Ferguson, Problem 3-1.

1-9. Ferguson, Problem 3-2.

1-10. Ferguson, Problem 3-3.

1-11. Ferguson, Problem 3-4.