

Name _____ Student ID _____

The exam is closed book and closed notes. You may use one $8\frac{1}{2} \times 11$ sheet of paper with formulas, etc. You may also use the handouts on “brand name distributions” and Greek letters. You may use a calculator.

Put all of your work on this test form (use the back if necessary). Show your work or give an explanation of your answer. No credit for answers with no indication of where they came from. Leave no undone integrals or means or variances in your answers, but other than that requirement there is no unique “correct” simplification. Any correct answer gets full credit, except as explicitly stated in questions.

Abbreviations used: asymptotic relative efficiency (ARE), distribution function (DF), probability density function (PDF), independent and identically distributed (IID).

The points for the questions total to 100. There are 6 pages and 5 problems.

1. [20 pts.] For the following data

-12.5 -6.5 -2.0 -1.0 2.0 26.0

- (a) Find the mean of the empirical distribution.

(b) Find the variance of the empirical distribution.

(c) Find a median of the empirical distribution.

(d) Find an upper quartile of the empirical distribution.

(e) Find the probability that X is strictly greater than zero when X is a random variable having the empirical distribution.

2. [20 pts.] The function

$$f_{\theta}(x) = \frac{\theta}{x^{\theta+1}}, \quad 1 < x < \infty$$

is a PDF (not a DF), where the parameter θ is any positive real number. Find the asymptotic distribution of the sample median of an IID sample having this distribution.

3. [20 pts.] Suppose X_1, X_2, \dots are IID $\text{NegBin}(r, p)$ random variables, where r is known and p is unknown and satisfies $0 < p < 1$, and suppose we are interested in estimating the parameter

$$\theta = \log(1 - p).$$

- (a) Find a method of moments estimator of θ .
- (b) Find the asymptotic normal distribution of your method of moments estimator.
- (c) Express the asymptotic variance in terms of the parameter θ only (no other parameters, no p , μ , or σ^2). Constants like r and n can appear.

- (d) Find the ARE of the sample mean and sample median of an IID sample from this distribution, both considered as estimators of the center of symmetry (if you do this right, the ARE will not depend on unknown parameters). Also state which estimator is better.
5. [20 pts.] Suppose X_1, X_2, \dots are IID from the $\text{Ber}(p)$ distribution, and suppose we are interested in the parameter

$$\theta = p(1 - p).$$

Find an asymptotic 95% confidence interval for θ , the endpoints of which are a function of \bar{X}_n only (no other statistics). Hint: the 0.975 quantile of the standard normal distribution is 1.9600.