Stat 5102 (Geyer) Spring 2016
Homework Assignment 8
Due Wednesday, November 9, 2016

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

8-1. Show that each of the following is an exponential family. Identify the natural parameter and natural statistic.

(a) The $\text{Poi}(\mu)$ family of distributions.

(b) The $\text{Exp}(\lambda)$ family of distributions.

(c) The $\text{Gam}(\alpha, \lambda)$ family of distributions with both parameters unknown. The natural parameter vector and natural statistic vector are both two-dimensional.

8-2. Suppose X is $\text{Poi}(\mu)$ and the prior distribution for μ is $\text{Gam}(\alpha, \lambda)$, where α and λ are hyperparameters. Find the posterior distribution for μ.

8-3. Suppose $X_1, \ldots, X_n$ are IID $\text{Gam}(\alpha, \lambda)$, where α is known and λ is unknown. Suppose the prior distribution for λ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters. Find the posterior distribution for λ.

8-4. Suppose $X_1, \ldots, X_n$ are IID $\text{Unif}(0, \theta)$ and the prior distribution for θ is $\text{Unif}(a, b)$, where a and b are hyperparameters. Find the PDF of the posterior distribution for θ. Under what conditions on $x_1, \ldots, x_n$, a, and b does the solution make no sense?

8-5. Suppose the distribution for data X is $\text{Geo}(p)$. Show that the beta family of distributions is conjugate.

8-6. Suppose $X_1, \ldots, X_n$ are IID $\mathcal{N}(\mu, 1/\lambda)$, where μ is known and λ is unknown. Find a brand-name family of distributions that is conjugate.

8-7. Suppose X is $\text{Geo}(p)$ and the prior distribution for p is $\text{Beta}(\alpha_1, \alpha_2)$, where α_1 and α_2 are hyperparameters. Find the posterior distribution for p.

8-8. Suppose $X_1, \ldots, X_n$ are IID $\mathcal{N}(\mu, 1/\lambda)$, where μ is known and λ is unknown. Suppose the prior distribution for λ is a distribution in the brand-name conjugate family of distributions found in problem 8-6. Find the posterior distribution for λ.

8-9. Suppose the situation is the same as in problem 8-8. Find the posterior distribution for $\sigma = \sqrt{1/\lambda}$. **Hint:** change-of-variable formula.

8-10. Suppose $X_1, \ldots, X_n$ are IID $\text{Exp}(\lambda)$.

(a) Suppose the prior distribution for λ is flat (an improper prior). Find the posterior distribution for λ.

(b) Suppose the prior distribution for λ is proportional to λ^{-1} (an improper prior). Find the posterior distribution for λ.

Review Problems from Previous Tests

8-11. Suppose $X_1, \ldots, X_n$ are IID $\text{Exp}(\lambda)$, and suppose the prior distribution for λ is $\text{Gam}(\alpha_0, \lambda_0)$, where α_0 and λ_0 are hyperparameters. Find the posterior distribution for λ.

8-12. Suppose X is $\text{Poi}(\mu)$. We have only one observation. And suppose the prior distribution for μ is proportional to $\mu^{-1/2}$, an improper prior.

(a) Find the posterior distribution for μ.

(b) For what values of the data x does your answer to part (a) make sense?