

Stat 5102 (Geyer) Spring 2016
Homework Assignment 3
Due Wednesday, September 28, 2016

Solve each problem. Explain your reasoning. No credit for answers with no explanation. If the problem is a proof, then you need words as well as formulas. Explain why your formulas follow one from another.

3-1. Show that the family of $\text{Gam}(\alpha, \lambda)$ distributions with α known and λ unknown, so the parameter space is

$$\{\lambda \in \mathbb{R} : \lambda > 0\}$$

is a scale family.

3-2. Suppose S_n^2 is the sample variance calculated from an IID normal random sample of size n .

- (a) Calculate the bias of S_n as an estimator of the population standard deviation σ .
- (b) Find the constant a such that aS_n has the smallest mean square error as an estimator of σ .

3-3. Suppose U and V are statistics that are independent random variables and both are unbiased estimators of a parameter θ . Write $\text{var}(U) = \sigma_U^2$ and $\text{var}(V) = \sigma_V^2$, and define another statistic $T = aU + (1 - a)V$ where a is an arbitrary but known constant.

- (a) Show that T is an unbiased estimator of θ .
- (b) Find the a that gives T the smallest mean square error.

3-4. The slides don't give any examples of estimators that are *not* consistent. Give an example of an inconsistent estimator of the population mean.

3-5. If $X \sim \text{Bin}(n, p)$, show that $\hat{p}_n = X/n$ is a consistent and asymptotically normal estimator of p , and give the asymptotic distribution of \hat{p}_n .

3-6. If X_1, X_2, \dots are IID from a distribution having a variance σ^2 , show that both V_n and S_n^2 are consistent estimators of σ^2 .

3-7. Suppose X_1, X_2, \dots are IID $\text{Geo}(p)$.

- (a) Find a method of moments estimator for p .
- (b) Find the asymptotic normal distribution of your estimator.

3-8. Suppose X_1, X_2, \dots are IID $\text{Beta}(\alpha, 2)$.

- (a) Find a method of moments estimator for α .
- (b) Find the asymptotic normal distribution of your estimator.

3-9. Let X_1, X_2, \dots, X_n be an i. i. d. sample from a $\text{Beta}(\theta, \theta)$ model, where θ is an unknown parameter. Find a method of moments estimator of θ .

Review Problems from Previous Tests

3-10. For the following data

1.5 2.0 2.5 3.0 4.5

- (a) Find the mean of the empirical distribution.
- (b) Find the median of the empirical distribution.
- (c) Find $P_n(X \leq 3)$ under the empirical distribution.
- (d) Find the 0.25 quantile of the empirical distribution.

3-11. Find the asymptotic distribution of the sample median of an IID sample from the $\text{Exp}(\lambda)$ distribution.

3-12. Suppose X_1, X_2, \dots are IID $\text{NegBin}(r, p)$, where r is known and p is unknown.

- (a) Find a method of moments estimator for p .
- (b) Find the asymptotic normal distribution of your estimator.