

Stat 5101 (Geyer) Spring 2022  
Homework Assignment 1  
Due Wednesday, January 26, 2022

Solve each problem. Explain your reasoning. No credit for answers with no explanation.

**1-1.** For each of the following functions  $h$  either determine a constant  $c$  such that  $c \cdot h$  — that is, the function  $x \mapsto c \cdot h(x)$  — is a PMF or determine that no such constant exists.

- (a) the identity function on the set  $\{0, 1, 2\}$ .
- (b) the identity function on the set  $\{-2, -1, 0, 1, 2\}$ .
- (c) the constant function  $x \mapsto 1$  on the set  $\{0, 1, 2\}$ .
- (d) the constant function  $x \mapsto 1$  on the set  $\{-2, -1, 0, 1, 2\}$ .
- (e) the function  $x \mapsto x^2$  on the set  $\{0, 1, 2\}$ .
- (f) the function  $x \mapsto x^2$  on the set  $\{-2, -1, 0, 1, 2\}$ .
- (g) the function  $x \mapsto x^3$  on the set  $\{0, 1, 2\}$ .
- (h) the function  $x \mapsto x^3$  on the set  $\{-2, -1, 0, 1, 2\}$ .

**1-2.** Suppose  $X$  is a random variable having the discrete uniform distribution on the sample space  $\{1, 2, 3, 4, 5, 6\}$ .

- (a) Determine  $\Pr(X < 4)$ .
- (b) Determine  $\Pr(X \leq 4)$ .
- (c) Determine  $\Pr(6 < X < 10)$ .

**1-3.** Suppose  $X$  is a random variable having PMF

$$f(x) = \frac{x}{21}, \quad x = 1, 2, 3, 4, 5, 6.$$

- (a) Determine  $E(X)$ .
- (b) Determine  $E(X^2)$ .
- (c) Determine  $E\{(X - 3)^2\}$ .

**1-4.** Suppose  $X$  is a  $\text{Ber}(p)$  random variable.

(a) Show that  $E(X^k) = p$  for all positive integers  $k$ .

(b) Determine  $E\{(X - p)^2\}$ .

(c) Determine  $E\{(X - p)^3\}$ .

**1-5.** Determine the set of real numbers  $\theta$  such that

$$f_{\theta}(x) = \begin{cases} \theta, & x = x_1 \\ \theta^2, & x = x_2 \\ 1 - \theta - \theta^2, & x = x_3 \end{cases}$$

is a PMF on the sample space  $\{x_1, x_2, x_3\}$ .