Computing Primer for Applied Linear Regression, Third Edition

Using SAS

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Home Website: www.stat.umn.edu/alr

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0 Introduction

This computer primer supplements the book *Applied Linear Regression* (ALR), third edition, by Sanford Weisberg, published by John Wiley & Sons in 2005. It shows you how to do the analyses discussed in ALR using one of several general-purpose programs that are widely available throughout the world. All the programs have capabilities well beyond the uses described here. Different programs are likely to suit different users. We expect to update the primer periodically, so check `www.stat.umn.edu/alr` to see if you have the most recent version. The versions are indicated by the date shown on the cover page of the primer.

Our purpose is largely limited to using the packages with ALR, and we will not attempt to provide a complete introduction to the packages. If you are new to the package you are using you will probably need additional reference material.

There are a number of methods discussed in ALR that are not (as yet) a standard part of statistical analysis, and some methods are not possible without writing your own programs to supplement the package you choose. *The exceptions to this rule are R and S-Plus. For these two packages we have written functions you can easily download and use for nearly everything in the book.*

Here are the programs for which primers are available.

R is a *command line* statistical package, which means that the user types a statement requesting a computation or a graph, and it is executed immediately. You will be able to use a package of functions for R that will let you use all the methods discussed in ALR; we used R when writing the book.

R also has a programming language that allows automating repetitive tasks. R is a favorite program among academic statisticians because it is free, works on Windows, Linux/Unix and Macintosh, and can be used in a great variety of problems. There is also a large literature developing on using R for statistical problems. The main website for R is `www.r-project.org`. From this website you can get to the page for downloading R by clicking on the link for `CRAN`, or, in the US, going to `cran.us.r-project.org`.

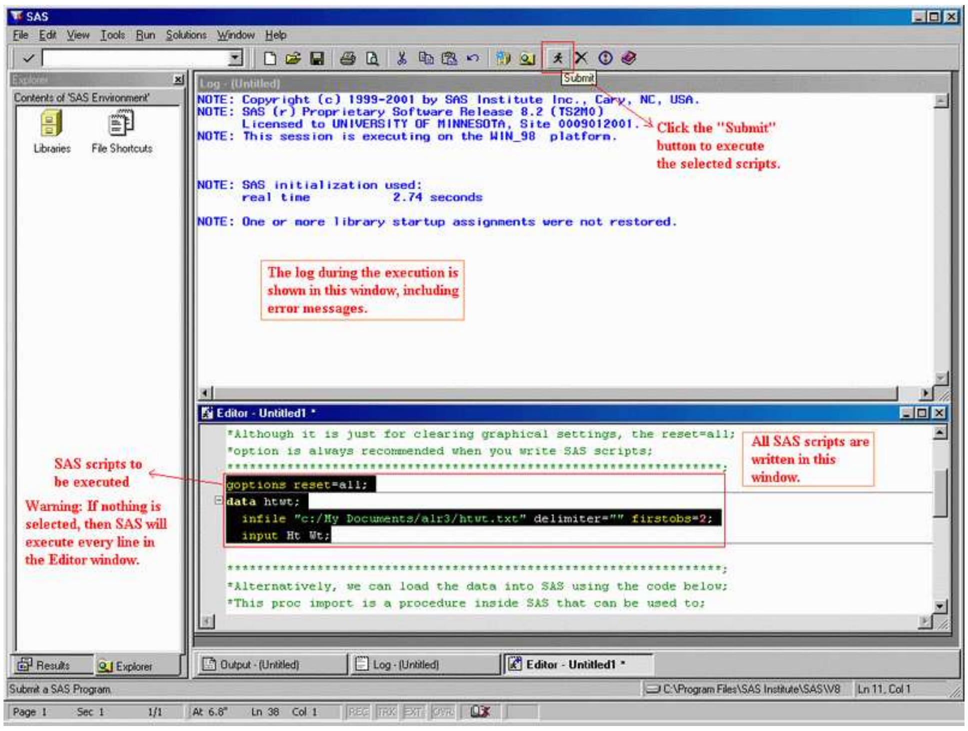
Documentation is available for R on-line, from the website, and in several books. We can strongly recommend two books. The book by Fox (2002) provides a fairly gentle introduction to R with emphasis on regression. We will from time to time make use of some of the functions discussed in Fox's book that are not in the base R program. A more comprehensive introduction to R is Venables and Ripley (2002), and we will use the notation VR[3.1], for example, to refer to Section 3.1 of that book. Venables and Ripley has more computerese than does Fox's book, but its coverage is greater and you will be able to use this book for more than linear regression. Other books on R include Verzani (2005), Maindonald and Braun (2002), Venables and Smith (2002), and Dalgaard (2002). We used R Version 2.0.0 on Windows and Linux to write the package. A new version of R is released twice a year, so the version you use will probably be newer. If you have a fast internet connection, downloading and upgrading R is easy, and you should do it regularly.

S-Plus is very similar to R, and most commands that work in R also work in S-Plus. Both are variants of a statistical language called "S" that was written at Bell Laboratories before the breakup of AT&T. Unlike R, S-Plus is a commercial product, which means that it is not free, although there is a free student version available at `elms03.e-academy.com/splus`. The website of the publisher is `www.insightful.com/products/splus`. A library of functions very similar to those for R is also available that will make S-Plus useful for all the methods discussed in ALR.

S-Plus has a well-developed graphical user interface or GUI. Many new users of S-Plus are likely to learn to use this program through the GUI, not through the command-line interface. In this primer, however, we make no use of the GUI.

If you are using S-Plus on a Windows machine, you probably have the manuals that came with the program. If you are using Linux/Unix, you may not have the manuals. In either case the manuals are available online; for Windows see the Help → Online Manuals, and for Linux/Unix use

```
> cd ‘Splus SHOME‘/doc
```



```
NOTE: Copyright (c) 1999-2001 by SAS Institute Inc., Cary, NC, USA.
NOTE: SAS (r) Proprietary Software Release 8.2 (TS2M0)
      Licensed to UNIVERSITY OF MINNESOTA, Site 0009012001.
NOTE: This session is executing on the WIN_98  platform.


NOTE: SAS initialization used:
      real time           2.74 seconds

NOTE: One or more library startup assignments were not restored.
```

Click the "Submit" button to execute the selected scripts.

The log during the execution is shown in this window, including error messages.

```
*Although it is just for clearing graphical settings, the reset=all;
*option is always recommended when you write SAS scripts;
*******************************************************************;
goptions reset=all;
data htwt;
  infile "c:/My Documents/alr3/htwt.txt" delimiter="" firstobs=2;
  input Ht Wt;

*******************************************************************;
*Alternatively, we can load the data into SAS using the code below;
*This proc import is a procedure inside SAS that can be used to;
```

All SAS scripts are written in this window.

SAS scripts to be executed

Warning: If nothing is selected, then SAS will execute every line in the Editor window.

Fig. 0.1 SAS windows.

```
> ls
```

and see the pdf documents there. Chambers and Hastie (1993) provides the basics of fitting models with S languages like S-Plus and R. For a more general reference, we again recommend Fox (2002) and Venables and Ripley (2002), as we did for R. We used S-Plus Version 6.0 Release 1 for Linux, and S-Plus 6.2 for Windows. Newer versions of both are available.

SAS is the largest and most widely distributed statistical package in both industry and education. SAS also has a GUI. When you start SAS, you get the collection of Windows shown in Figure 0.1. While it is possible to do *some* data analysis using the SAS GUI, the strength of this program is in the ability to write SAS programs, in the editor window, and then submit them for execution, with output returned in an output window. We will therefore view SAS as a *batch* system, and concentrate mostly on writing SAS commands to be executed. The website for SAS is `www.sas.com`.

SAS is very widely documented, including hundreds of books available through amazon.com or from the SAS Institute, and extensive on-line documentation. Muller and Fetterman (2003) is dedicated particularly to regression. We used Version 9.1 for Windows. We find the on-line documentation that accompanies the program to be invaluable, although learning to read and understand SAS documentation isn't easy.

Although SAS is a programming language, adding new functionality can be very awkward and require long, confusing programs. These programs could, however, be turned into SAS *macros* that could be reused over and over, so in principle SAS could be made as useful as R or S-Plus. We have not done this, but would be delighted if readers would take on the challenge of writing macros for methods that are awkward with SAS. Anyone who takes this challenge can send us the results (sandy@stat.umn.edu) for inclusion in later revisions of the primer.

We have, however, prepared *script files* that give the programs that will produce all the output discussed in this primer; you can get the scripts from `www.stat.umn.edu/alr`.

JMP is another product of SAS Institute, and was designed around a clever and useful GUI. A student version of JMP is available. The website is `www.jmp.com`. We used JMP Version 5.1 on Windows.

Documentation for the student version of JMP, called JMP-In, comes with the book written by Sall, Creighton and Lehman (2005), and we will write JMP-START[3] for Chapter 3 of that book, or JMP-START[P360] for page 360. The full version of JMP includes very extensive manuals; the manuals are available on CD only with JMP-In. Fruend, Littell and Creighton (2003) discusses JMP specifically for regression.

JMP has a scripting language that could be used to add functionality to the program. We have little experience using it, and would be happy to hear from readers on their experience using the scripting language to extend JMP to use some of the methods discussed in ALR that are not possible in JMP without scripting.

SPSS evolved from a batch program to have a very extensive graphical user interface. In the primer we use only the GUI for SPSS, which limits the methods that are available. Like SAS, SPSS has many sophisticated tools for data base management. A student version is available. The website for SPSS is `www.spss.com`. SPSS offers hundreds of pages of documentation, including SPSS (2003), with Chapter 26 dedicated to regression models. In mid-2004, amazon.com listed more than two thousand books for which "SPSS" was a keyword. We used SPSS Version 12.0 for Windows. A newer version is available.

This is hardly an exhaustive list of programs that could be used for regression analysis. If your favorite package is missing, please take this as a challenge: try to figure out how to do what is suggested in the text, and write your own primer! Send us a PDF file (sandy@stat.umn.edu) and we will add it to our website, or link to yours.

One program missing from the list of programs for regression analysis is Microsoft's spreadsheet program Excel. While *a few* of the methods described in the book can be computed or graphed in Excel, most would require great endurance and patience on the part of the user. There are many add-on statistics programs for Excel, and one of these may be useful for comprehensive regression analysis; we don't know. If something works for you, please let us know!

A final package for regression that we should mention is called Arc. Like R, Arc is free software. It is available from `www.stat.umn.edu/arc`. Like JMP and SPSS it is based around a graphical user interface, so most computations are done via point-and-click. Arc also includes access to a complete computer language, although the language, lisp, is considerably harder to learn than the S or SAS languages. Arc includes all the methods described in the book. The use of Arc is described in Cook and Weisberg (1999), so we will not discuss it further here; see also Weisberg (2005).

0.1 ORGANIZATION OF THIS PRIMER

The primer often refers to specific problems or sections in ALR using notation like ALR[3.2] or ALR[A.5], for a reference to Section 3.2 or Appendix A.5, ALR[P3.1] for Problem 3.1, ALR[F1.1] for Figure 1.1, ALR[E2.6] for an equation and ALR[T2.1] for a table. Reference to, for example, "Figure 7.1," would refer to a figure in this primer, not to ALR. Chapters, sections, and homework problems are numbered in this primer as they are in ALR. Consequently, the section headings in primer refers to the material in ALR, and not necessarily the material in the primer. Many of the sections in this primer don't have any material because that section doesn't introduce any new issues with regard to computing. The index should help you navigate through the primer.

There are four versions of this primer, one for R and S-Plus, and one for each of the other packages. All versions are available for free as PDF files at `www.stat.umn.edu/alr`.

Anything you need to type into the program will always be in `this font`. Output from a program depends on the program, but should be clear from context. We will write File to suggest selecting the menu called "File," and Transform → Recode to suggest selecting an item called "Recode" from a menu called "Transform." You will sometimes need to push a button in a dialog, and we will write "push OK" to mean "click on the button marked 'OK'." For non-English versions of some of the programs, the menus may have different names, and we apologize in advance for any confusion this causes.

Table 0.1 The data file `htwt.txt`.

```
Ht Wt
169.6 71.2
166.8 58.2
157.1 56
181.1 64.5
158.4 53
165.6 52.4
166.7 56.8
156.5 49.2
168.1 55.6
165.3 77.8
```

0.2 DATA FILES

0.2.1 Documentation

Documentation for nearly all of the data files is contained in ALR; look in the index for the first reference to a data file. Separate documentation can be found in the file `alr3data.pdf` in PDF format at the web site `www.stat.umn.edu/alr`.

The data are available in a *package* for R, in a *library* for S-Plus and for SAS, and as a directory of files in special format for JMP and SPSS. In addition, the files are available as plain text files that can be used with these, or any other, program. Table 0.1 shows a copy of one of the smallest data files called `htwt.txt`, and described in ALR[P3.1]. This file has two variables, named *Ht* and *Wt*, and ten cases, or rows in the data file. The largest file is `wm5.txt` with 62,040 cases and 14 variables. This latter file is so large that it is handled differently from the others; see Section 0.2.4.

A few of the data files have missing values, and these are generally indicated in the file by a place-holder in the place of the missing value. For example, for R and S-Plus, the placeholder is `NA`, while for SAS it is a period "." Different programs handle missing values a little differently; we will discuss this further when we get to the first data set with a missing value in Section 4.5.

0.2.2 SAS data library

The data for use with SAS are provided in a special SAS format, or as plain data files. Instructions for getting and installing the SAS library are given on the SAS page at `www.stat.umn.edu/alr`.

If you follow the directions on the web site, you will create a data library called `alr3` that will always be present when you start SAS. Most procedures in SAS have a required `data` keyword that tells the program where to find the data to be used. For example, the simple SAS program

```
proc means data = alr3.heights;
```

```
run;
```

tells SAS to run the means procedure using the data set `heights` in the library `alr3`. To run this program, you type it into the editor window, and then select and submit it; see Section 0.4.1.

0.2.3 Getting the data in text files

You can download the data as a directory of plain text files, or as individual files; see `www.stat.umn.edu/alr/data`. *Missing values on these files are indicated with a* `?`. *If your program does not use this missing value character, you may need to substitute a different character using an editor.*

0.2.4 An exceptional file

The file `wm5.txt` is not included in any of the compressed files, or in the libraries. This one file is nearly five megabytes long, requiring as much space as all the other files combined. If you need this file, for ALR[P10.12], you can download it separately from `www.stat.umn.edu/alr/data`.

0.3 SCRIPTS

For R, S-Plus, and SAS, we have prepared *script files* that can be used while reading this primer. For R and S-Plus, the scripts will reproduce nearly every computation shown in ALR; indeed, these scripts were used to do the calculations in the first place. For SAS, the scripts correspond to the discussion given in this primer, but will not reproduce everything in ALR. The scripts can be downloaded from `www.stat.umn.edu/alr` for R, S-Plus or SAS.

Although both JMP and SPSS have scripting or programming languages, we have not prepared scripts for these programs. Some of the methods discussed in ALR are not possible in these programs without the use of scripts, and so we encourage readers to write scripts in these languages that implement these ideas. Topics that require scripts include bootstrapping and computer intensive methods, ALR[4.6]; partial one-dimensional models, ALR[6.4], inverse response plots, ALR[7.1, 7.3], multivariate Box-Cox transformations, ALR[7.2], Yeo-Johnson transformations, ALR[7.4], and heteroscedasticity tests, ALR[8.3.2]. There are several other places where usability could be improved with a script.

If you write scripts you would like to share with others, let me know (sandy@stat.umn.edu) and I'll make a link to them or add them to the website.

0.4 THE VERY BASICS

Before you can begin doing any useful computing, you need to be able to read data into the program, and after you are done you need to be able to save and print output and graphs. All the programs are a little different in how they handle input and output, and we give some of the details here.

0.4.1 Reading a data file

Reading data into a program is surprisingly difficult. We have tried to ease this burden for you, at least when using the data files supplied with ALR, by providing the data in a special format for each of the programs. There will come a time when you want to analyze real data, and then you will need to be able to get your data into the program. Here are some hints on how to do it.

SAS If you have created the SAS library called `alr3` as suggested in Section 0.2.2, then all the data files are ready for your use in SAS procedures. For example, the data in `htwt.txt` is available by referring to the dataset `alr3.htwt`. If you did not create the library, you can read the data from the *plain text files* provided into SAS using the menu item File → Import data file.

If you select this menu item, the SAS import wizard will guide you through the process of importing the data. In the first window, select the type of file you want to import. If you want to import one of the files supplied with ALR such as `htwt.txt`, select "Delimited file (*.*)" from the popup menu. Push NEXT, and on the next screen browse to find the file on your disk. Use the OPTIONS button on this screen to check the settings for this file. For all the data files that are provided, the delimiter is a space, the first row of data is row 2, and "Get variable names from the first row" should be checked. Push NEXT. On the third screen, you need to assign a name and a library for this data set. For example, if you keep the default library `work` and name the data set as `htwt`, then in SAS programs you will refer to this data set as `work.htwt`. The `work` library is special because files imported into it are temporary, and will disappear when you close SAS. If you import the file to any other library such as `sasuser`, then the file is permanent, and will be available when you restart SAS.

After selecting a library and name, push FINISH. If you push NEXT, you will be given the opportunity to save the SAS program generated by the wizard as a file. The code that is saved looks like this:

```
PROC IMPORT OUT= WORK.htwt
            DATAFILE= "z:\working\alr3-data\htwt.txt"
            DBMS=DLM REPLACE;
     DELIMITER='20'x;
     GETNAMES=YES;
     DATAROW=2;
```

```
RUN;
```

Using the wizard is equivalent to writing this SAS program and then executing it. Like most SAS programs: (1) all statements end with a semicolon "`;`", (2) the program consists of blocks that begin with a `proc` to start a procedure and ending with a `run;` to execute it, and (3) several statements are included into the procedure call, each ending with a semicolon. There are a few exceptions to these rules that we will encounter later.

SAS is *not case-sensitive*, meaning that you can type programs in capitals, as shown above, using lower-case letters, or any combination of them you like.

0.4.2 Saving text output and graphs

All the programs have many ways of saving text output and graphs. We will make no attempt to be comprehensive here.

SAS Some SAS procedures produce a lot of output. SAS output is by default centered on the page, and assumes that you have paper with 132 columns. You can, however, change these defaults to other values. For example, the following line at the beginning of a SAS program

```
options nocenter linesize=75 pagesize=66;
```

will produce left-justified output, assuming 75 columns on a page, and a page length of 66 lines.

To save numerical output, click in the output window, and select File → Save As. Alternatively, select the output you want and copy and paste all the output to a word processing document or editor. If you use a word processor like Microsoft Word, be sure to use a monospaced font like Courier New so the columns line up properly. To save a graph, click it and then select File → Export as Image. In the dialog to save the graph, we recommend that you save files using "gif" format; otherwise the graphics files will be very large. You can then print the graph, or import it into a word processing document. We do not recommend copying the graph to the clipboard and pasting it into a word processing document.

0.4.3 Normal, F, t and χ^2 tables

ALR does not include tables for looking up critical values and significance levels for standard distributions like the t, F and χ^2. Although these values can be computed with any of the programs we discuss in the primers, doing so is easy only with R and S-Plus. Also, the computation is fairly easy with Microsoft Excel. Table 0.2 shows the functions you need using Excel.

SAS SAS allows computing the computing critical values and significance levels for a very long list of distributions. Two functions are used, called `CDF`

Table 0.2 Functions for computing p-values and critical values using Microsoft Excel. The definitions for these functions are not consistent, sometimes corresponding to two-tailed tests, sometimes giving upper tails, and sometimes lower tails. Read the definitions carefully. The algorithms used to compute probability functions in Excel are of dubious quality, but for the purpose of determining p-values or critical values, they should be adequate; see Knüsel (2005) for more discussion.

Function	What it does
`normsinv(p)`	Returns a value q such that the area to the left of q for a standard normal random variable is p.
`normsdist(q)`	The area to the left of q. For example, `normsdist(1.96)` equals 0.975 to three decimals.
`tinv(p,df)`	Returns a value q such that the area to the left of $-\|q\|$ *and* the area to the right of $+\|q\|$ for a $t(\mathrm{df})$ distribution equals q. This gives the critical value for a two-tailed test.
`tdist(q,df,tails)`	Returns p, the area to the left of q for a $t(df)$ distribution if $tails = 1$, and returns the sum of the areas to the left of $-\|q\|$ and to the right of $+\|q\|$ if $tails = 2$, corresponding to a two-tailed test.
`finv(p,df1,df2)`	Returns a value q such that the area to the *right* of q on a $F(\mathrm{df}_1, \mathrm{df}_2)$ distribution is p. For example, `finv(.05,3,20)` returns the 95% point of the $F(3, 20)$ distribution.
`fdist(q,df1,df2)`	Returns p, the area to the *right* of q on a $F(\mathrm{df}_1, \mathrm{df}_2)$ distribution.
`chiinv(p,df)`	Returns a value q such that the area to the *right* of q on a $\chi^2(\mathrm{df})$ distribution is p.
`chidist(q,df)`	Returns p, the area to the *right* of q on a $\chi^2(\mathrm{df})$ distribution.

and `quantile`. These functions are used inside the data step in SAS, and so you need to write a short SAS program to get tabled values. For example, the following program returns the p-value from a $\chi^2(25)$ test with value of the test statistic equal to 32.5, and also the critical value at the 95% level for this test.

```
data qt;
  pval = 1-CDF('chisquare',32.5,25);
  critval = quantile('chisquare',.95,25);
  output;
proc print data=qt; run;
```

which returns the output

```
Obs      pval     critval
```

```
 1      0.14405     37.6525
```

giving the p-value of .14, and the critical value 37.6, for the test.

To get F or t significance levels, use functions similar to the following:

`1-CDF('F',value,df1,df2)` *F p-values*
`quantile('F',level,df1,df2)` *F critical values*
`1-CDF('t',value,df)` *t p-values*
`quantile('t',level,df)` *t critical values*

For more information, go to Index tab in SAS on-line help, and type either quantile or cdf.

0.5 ABBREVIATIONS TO REMEMBER

ALR refers to the textbook, Weisberg (2005). VR refers to Venables and Ripley (2002), our primary reference for R and S-Plus. JMP-START refers to Sall, Creighton and Lehman (2005), the primary reference for JMP. Information typed by the user looks like `this`. References to menu items looks like File or Transform → Recode. The name of a BUTTON to push in a dialog uses this font.

0.6 COPYRIGHT AND PRINTING THIS PRIMER

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1 Scatterplots and Regression

1.1 SCATTERPLOTS

A principal tool in regression analysis is the two-dimensional scatterplot. All statistical packages can draw these plots. We concentrate mostly on the basics of drawing the plot. Most programs have options for modifying the appearance of the plot. For these, you should consult documentation for the program you are using.

SAS Most of the graphics available in SAS are *static*, meaning that the user writes a SAS program that will draw the graph, but cannot then interact with the graph, for example to identify points. The exception is for graphs created using the *Interactive Data Analysis* tool obtained by choosing Solutions → Analysis → Interactive Data Analysis. We concentrate here on static graphics because they are generally drawn using SAS programs. However, since the standard graphing method is first to save the data to be plotted in a data set, and then use `proc gplot`, one could equally well use another procedure like interactive data analysis for the graphs.

A basic procedure for scatterplots is `proc gplot`. The following SAS code will give a graph similar to ALR[F1.1] using the `heights` data:

```
proc gplot data=alr3.heights;
  plot Dheight*Mheight;
run; quit;
```

As with most SAS examples in this primer, we assume that you have the data from ALR available as a SAS library called `alr3`; see Section 0.2.2. The

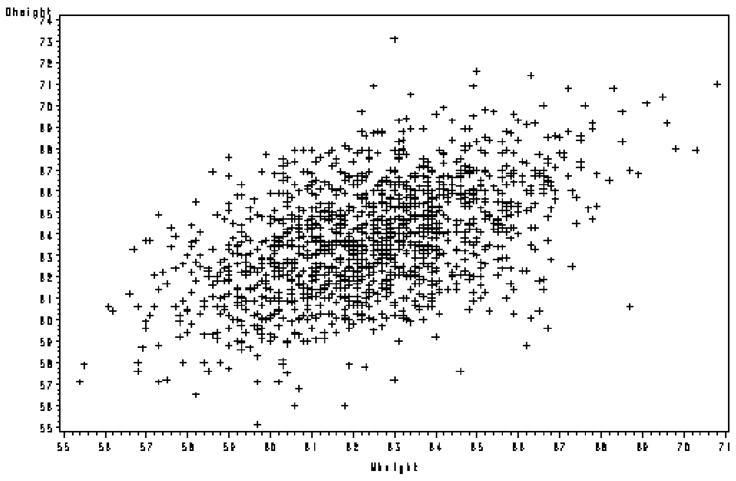


Fig. 1.1 SAS version of ALR[F1.1] with the `heights` data.

phrase `data=alr3.heights` specifies that the data will come from the data set called `heights` in the `alr3` library. The phrase `plot Dheight*Mheight` draws the plot with *Mheight* on the horizontal axis and *Dheight* on the vertical. `run;` tells SAS to execute the procedure, and `quit;` tells SAS to close the graph, meaning that it will not be enhanced by adding more points or lines. Simple graphs like this one require only a very short SAS program. The output from the above program is shown in Figure 1.1. We saved this file as a PostScript file because that is required by our typesetting program. SAS postscript files seem to be of low quality.

You can also use the menus to draw simple graphs like this one. Select Solutions → Analyze → Interactive data analysis, and from in the resulting window, select the library `alr3` and the data set `heights` in that library. The data will appear in a spreadsheet. Select Analyze → Scatter plot to draw this plot. An advantage to this method of drawing the plot is that the spreadsheet and the plot will be *linked*, meaning that selecting points on the plot will highlight the corresponding values in the spreadsheet. This can be useful for identifying unusual points.

Returning to `proc gplot`, you can add commands that will modify the appearance of the plot. These options must be added before the call to `proc gplot`. Continuing with ALR[F1.1],

```
goptions reset=all;
title ' ';
symbol v=dot c=black h=.1;
axis1 label=('Mheight');
axis2 label=(a=90 'Dheight');
proc gplot data=alr3.heights;
  plot Dheight*Mheight /haxis=axis1 haxis=55 to 75 hminor=0
```

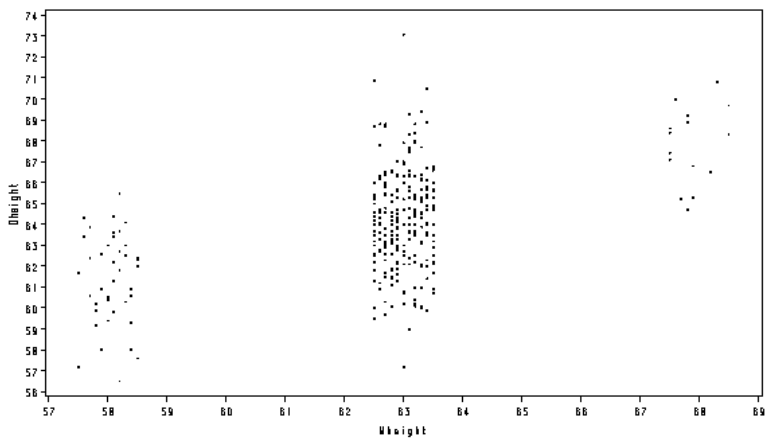


Fig. 1.2 ALR[F1.2] with the **heights** data.

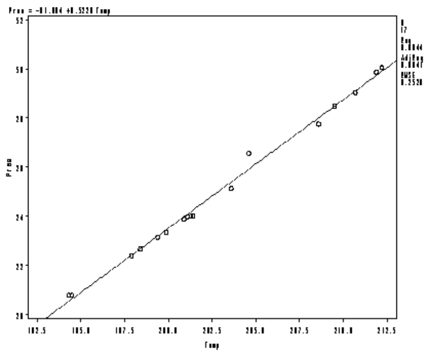
```
                        vaxis=axis2 vaxis=55 to 75 vminor=0;
run; quit;
```

We use the `goptions` command to reset all options to their default values so left-over options from a previous graph won't appear in the current one. The next four commands set the title, the plotting symbol, and the labels for the two axes. The vertical axis label has been rotated ninety degrees. In in `gplot` command, we have specified the `haxis` and `vaxis` keywords are set to `axis1` and `axis2` to get the variable labels. The remainder of the command concerns tick marks and to ensure that both axes extended from 55 to 75.

Similar SAS code to produce a version of ALR[F1.2] illustrates using the `where` statement:

```
goptions reset=all;
proc gplot data=alr3.heights;
  plot Dheight*Mheight /haxis=axis1 haxis=55 to 75 hminor=0
                        vaxis=axis2 vaxis=55 to 75 vminor=0;
  where (Mheight ge 57.5) & (Mheight le 58.5) |
        (Mheight ge 62.5) & (Mheight le 63.5) |
        (Mheight ge 67.5) & (Mheight le 68.5);
run; quit;
```

Inside `proc gplot`, specify the data set by `data=`*`dataset-name`*. The first argument *Dheight* will be the vertical axis variable, and the second argument *Mheight* will be on the horizontal axis. As with many SAS commands, options for a command like `plot` are added after a `/`. We let the horizontal axis be `axis1`, and the vertical axis be `axis2`. The `where` statement uses syntax similar to the C language. The logical operators `&` for *and* and | for *or* are used to specify ranges for plotting.


a. (a) OLS prediction.

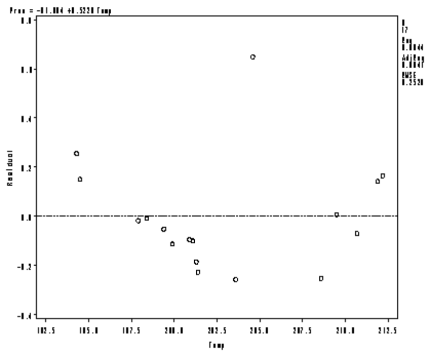

b. (b) OLS residual plot.

Fig. 1.3 SAS version of ALR[F1.3]. (sassupp104)

A few reminders:

1. All statements end with `;`.
2. Always use `goptions reset=all;` to remove all existing graphical setup before plotting.
3. Statements for changing graphical options like `symbol` have to be specified before `proc gplot` is called.
4. Nearly all SAS commands end with `run;`.

ALR[F1.3-1.4] includes both points and fitted lines. For ALR[F1.3A], we need to get the OLS fitted line. For ALR[F1.3B], we need to compute residuals. This can be done by calling `proc reg` in SAS, which also allows graphing:

```
goptions reset=all;
symbol v=circle h=1;
proc reg data=alr3.forbes;
  model Pressure=Temp;
  plot Pressure*Temp;
  plot residual.*Temp;
run;
```

We define `circle` as the plotting symbol in the `symbol` statement. For the `model` statement, the syntax is `model response=terms;`. The `plot` command in `proc reg` draws graphs of regression quantities, with the syntax `horizontal*vertical`. With simple regression, the plot of the response versus the single predictor adds the OLS line automatically. The second plot makes use of the keyword `residual.` to get the residuals. *Do not forget the dot after keywords!*

ALR[F1.5] includes both an OLS line and a line that joins the mean lengths at each age. Although this plot seems simple, the SAS program we wrote to obtain it is surprisingly difficult and not particularly intuitive. Here is the SAS, program:

```
proc loess data=alr3.wblake;
  model Length=Age /smooth=.1;
  ods output OutputStatistics=m1;
run;
proc reg data=alr3.wblake;
  model Length=Age;
  output out=m2 pred=ols;
run;
data m3; set alr3.wblake; set m1; set m2;
proc sort data=m3; by Age;
goptions reset=all;
symbol1 v=circle h=1 c=black l=1 i=join;
symbol2 v=circle h=1 c=black l=2 i=join;
symbol3 v=circle h=1 c=black;
axis2 label=(a=90 'Length');
proc gplot data=m3;
  plot ols*Age=1 Pred*Age=2 Length*Age=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run; quit;
```

The plot consists of three parts: the points, the OLS fitted line, and the line joining the means for each value of *Age*. We couldn't think of obvious way to get this last line, and so we used a trick. The *loess smoother*, discussed in ALR[A.5], can be used for this purpose by setting the smoothing parameter to be a very small number, like .1. The call shown to `proc loess` will compute the mean *Length* for each *Age* called `m1`. Confusingly, you use an `ods` statement, not `output`, to save the output from `proc lowess`. Next, `proc reg` is used to get the regression of *Length* on *Age*. The output is saved using the `output` statement, not `ods`. This output includes the input data and the predicted values in a new data set called `m2`.

We now have the information for joining the points in the data set `m1` and the data for fitting OLS in `m2`. The `data` statement combines them with `alr3.wblake` in another data set called `m3`. A few graphics commands are then used. The , `symbol2` and `symbol3` define plotting symbols; you can define `symbol4`, `symbol5` and so on. The option `i=join` tells SAS to connect the points using straight lines. The option `l=1` refers to a connection with solid line, and `l=2` refers to a connection with a dashed line. `proc sort` was used to arrange the data set by an ascending order (which is the default) of variable *Age*. This makes sure that we do not get a mess when we connect data points to get the OLS line. After we customize each `symbol`, we can write `ols*Age=1 Pred*Age=2 Length*Age=3` in the `plot` statement, which simply means that use the first customized `symbol` for the plot of *ols* versus *Age*, the second customized `symbol` for the plot of loess *Pred* versus *Age*, and the third customized `symbol` for the scatterplot of *Length* versus *Age*. `proc gplot` was called with the data set `m3`. The `overlay` option further modifies the axes of the plot.

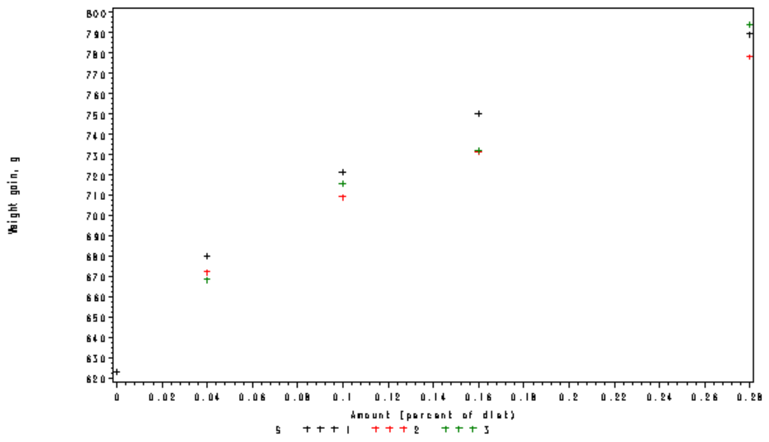


Fig. 1.4 SAS version of ALR[F1.7].

The only part ALR[F1.6] that is different from previous graphs is the horizontal line. Getting this line is surprisingly difficult. We recommend that you define `one=1` in the data step so that we can fit a linear regression of *Late* on *one*. All the fitted values for this regression will be equal to the overall average.

ALR[F1.7] is easier to obtain in SAS, because the variable S takes on values 1, 2, 3, which can be directly used in `plot` statement in `proc gplot`:

```
goptions reset=all;
proc gplot data=alr3.turkey;
  plot gain*A=S;
run; quit;
```

We write the `plot` statement as `plot` *y*x=z;*, which allows us to make a scatterplot of y versus x with separate symbol for each value of z variable.

1.2 MEAN FUNCTIONS

SAS ALR[F1.8] adds the line `y=x` to the plot. We fit a linear regression model first, save the predicted values to a data set, which is called `m1` here, using the `output` statement. After this, we call the `proc gplot` to get a version of ALR[F1.8]. The SAS code is given below:

```
proc reg data=alr3.heights;
  model Dheight=Mheight;
  output out=m1 predicted=Dhat;
run;
goptions reset=all;
proc gplot data=m1;
  plot (Dhat Mheight Dheight)*Mheight /overlay ;
```

```
run; quit;
```

1.3 VARIANCE FUNCTIONS

1.4 SUMMARY GRAPH

1.5 TOOLS FOR LOOKING AT SCATTERPLOTS

SAS `proc loess` can be used to add a *loess* smoother to a graph. The following produces a version of ALR[F1.10]:

```
proc reg data=alr3.heights;
  model Dheight=Mheight;
  output out=m1 predicted=Dhat;
run;
proc loess data=alr3.heights;
  model Dheight=Mheight /smooth=.6;
  ods output OutputStatistics=myout;
run;
data myout; set myout; set m1;
proc sort data=myout; by Mheight; run;
goptions reset=all;
proc gplot data=myout;
  plot (Dhat Pred DepVar)*Mheight /overlay;
run; quit;
```

This procedure produces *loess* estimates given a smoothing parameter, which is 0.6 here (specified using `smooth`). *The smoothing parameter must be strictly positive and less than one.* The statement `ods output` is used to produce procedure-specific output. We have saved the output in a data set called `myout`. After combining `myout` from `proc loess` and the output `m1` from OLS fit, we sort the data by *Mheight*, the variable to be plotted on the horizontal axis. As we have mentioned before, this is simply for visual clarification since we are going to connect the *loess* estimates to get a *loess* curve. *DepVar* is just the dependent variable *Dheight*, in a different name in the output data `myout`. *Pred* is the *loess* estimate. The `quit;` statement stops procedures called before it.

1.6 SCATTERPLOT MATRICES

SAS This is the first example we have encountered that requires transformation of some of the variables to be graphed. This is done by creating a new data set that consists of the old variables plus the transformed variables. The `data` step in SAS for transformations.

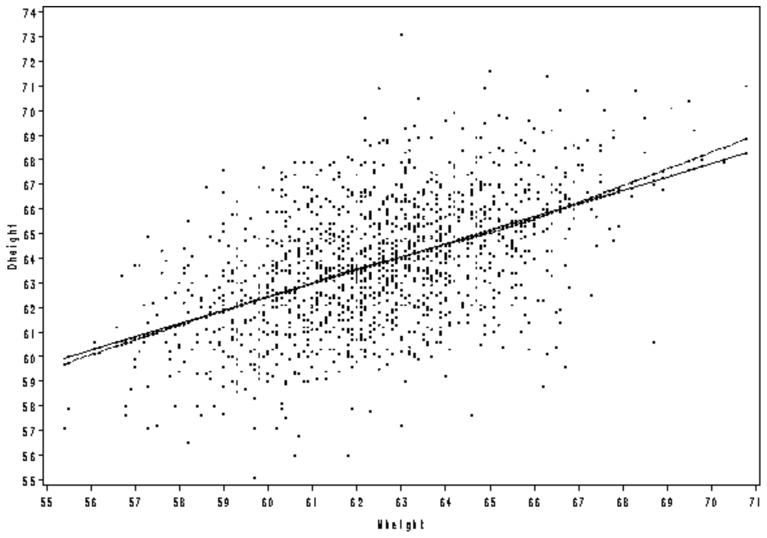


Fig. 1.5 SAS version of ALR[F1.10].

Scatterplot matrices are obtained using `proc insight`. For the `fuel2001` data, here is the program:

```
data m1;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  logMiles=log2(Miles);
goptions reset=all;
proc insight data=m1;
  scatter Tax Dlic income logMiles Fuel
         *Tax Dlic income logMiles Fuel;
run;
```

In the `data` step, we created a new data set called `m1` that includes all of `alr3.fuel2001` from the `set` statement, and the three additional variables computed from transformations; note the use of `log2` for base-two logarithms. We called `proc insight` using this new data set.

Solutions $\rightarrow$ Analysis $\rightarrow$ Interactive Data Analysis also starts `proc insight`. You can also transform variables from within this procedure.

Graphs created with `proc insight` are interactive. If you click the mouse on a point in this graph, the point will he highlighted in all frames of the scatterplot matrix, and its case number will be displayed on the graph.

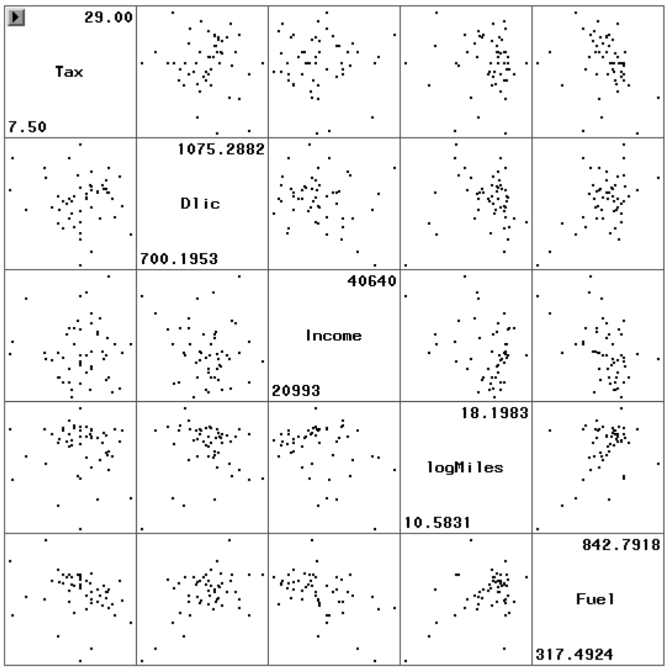


Fig. 1.6 SAS scatterplot matrix for the `fuel2001` data.

Problems

1.1. Boxplots would be useful in a problem like this because they display level (median) and variability (distance between the quartiles) simultaneously.

SAS SAS has a `proc boxplot` procedure. You can follow the generic syntax as follows:

```
proc boxplot data=alr3.wblake;
  plot Length*Age;
run;
```

The above SAS code means that a separate boxplot of *Length* will be provided at each value of *Age*.

As an alternative, you can use the SAS *Interactive Data Analysis* tool to obtain boxplots. After you launch it with a data set, you can choose Analyze → Box Plot/Mosaic Plot(Y) from the menu. Then you click on *Length* once and click the `Y` button once. Similarly, you can move the variable *Age* to the box under the `X` button. After you click the `OK` button, a series of boxplots of *Length* for each value of *Age* are printed on the screen.

Just a reminder: Even if you choose to use SAS *Interactive Data Analysis* tool, you must have the data available in advance, either using a SAS data step, or using a data file from a library.

To get the standard deviation at each value of *Age*, you can call `proc means` or `proc summary` (which is exactly the same as `proc means` except that it does not show output) and put *Age* in the `class` statement. The keyword for retrieving standard deviation is `std`.

1.2.

SAS To change the size of an image in SAS, you can move you mouse cursor to the right-bottom edge of the graph window. When the cursor becomes a double arrow, click and hold the left button of your mouse, then drag the edge to any size.

1.3.

SAS In SAS, `log2(Fertility)` returns the base-two logarithms. If you need transformations, use the procedure outlined in Section 1.6 of the primer.

2 Simple Linear Regression

2.1 ORDINARY LEAST SQUARES ESTIMATION

All the computations for simple regression depend on only a few summary statistics; the formulas are given in the text, and in this section we show how to do the computations step–by-step. All computer packages will do these computations automatically, as we show in Section 2.6.

2.2 LEAST SQUARES CRITERION

SAS The procedure `proc means` can be used to get summary statsitics; see Table 2.1.

```
proc means data=alr3.forbes;
  var Temp Lpres;
run;
```

Table 2.1 `proc means` output with the `forbes` data.

```
The MEANS Procedure

Variable     N            Mean         Std Dev         Minimum         Maximum
Temp        17     202.9529412       5.7596786     194.3000000     212.2000000
Lpres       17     139.6052941       5.1707955     131.7900000     147.8000000
```

This chapter illustrates the computations that are usually hidden by regression programs. You can follow along with the calculations with the interactive matrix language in SAS, called `proc iml`.

```
data f;
  set alr3.forbes ;
  one=1;
proc iml;
use f;
read all var {one Temp Pressure Lpres}
     where (Temp^=. & Pressure^=. & Lpres^=.) into X;
size=nrow(X); W=X`*X;
Tempbar=W[1,2]/size; Lpresbar=W[1,4]/size; Y=j(size,2,0);
Y[,1]=X[,2]-Tempbar; Y[,2]=X[,4]-Lpresbar;
fcov=Y`*Y; RSS=fcov[2,2]-fcov[1,2]**2/fcov[1,1];
sigmahat2=RSS/(size-2); sigmahat=sqrt(sigmahat2);
b1=fcov[1,2]/fcov[1,1];
b0=Lpresbar-b1*Tempbar;
title 'PROC IML results';
print Tempbar Lpresbar fcov b1 b0 RSS sigmahat2;
create forbes_stat var{Tempbar Lpresbar fcov b1 b0 RSS sigmahat2};
append var{Tempbar Lpresbar fcov b1 b0 RSS sigmahat2};
close forbes_stat;
quit;
```

Table 2.2 `proc iml` results with the `forbes` data.

```
                          PROC IML results
  TEMPBAR  LPRESBAR      FCOV                  B1        B0       RSS SIGMAHAT2
202.95294 139.60529 530.78235 475.31224 0.8954937 -42.13778
2.1549273 0.1436618
                    475.31224 427.79402
```

We start by creating a new data set called `f` that has all the variables in `alr3.forbes` plus a new variable called `one` that repeats the value 1 for all cases in the data set, so it is a vector of length 17.

The keywords used in `proc iml` include `use`, `read all var`, `where` and `into`. These load all the variables of interest into the procedure so that we can compute various summary statistics. The variables in the braces are collected into a new matrix variable we called X, whose columns represent the variables in the order they are entered. The `where` statement guarantees that all calculations will be done on non-missing values only. "`^=`" stands for "not equal to", and is equivalent to the operator `ne` (*not equal*) outside `proc iml`. The operator ``X`*X`` denotes a matrix product $X'X$. *Use the single-quote located at the upper left corner of the computer keyboard, not the usual one, to denote the transpose.* An alternative way of specifying the transpose of a matrix X is by `t(X)`. The function `j(size,2,0)` creates a matrix of dimension *size by 2* of all zeroes. The operator "**2" is for exponentiating, here squaring. The `print` statement asks SAS to display some results, and the `create` and `append` statements save some of the variables into a data set called `forbes_stat`. The `create` statement defines a structure of the data set called `forbes_stat`, and the `append` statement fills in values. If some variables do not have the same length, then "." will be used in the new data set for missing values. The `close` statement simply denotes closing the data set.

The SAS code gives the sample means, the sample covariance matrix, regression coefficient estimates, *RSS* and variance estimate; see Table 2.2.

2.3 ESTIMATING σ^2

SAS From `proc iml`, we can easily get the variance estimate as `sigmahat2` in the output of `proc iml` above. RSS is also given above from `proc iml`. See the last section.

2.4 PROPERTIES OF LEAST SQUARES ESTIMATES

2.5 ESTIMATED VARIANCES

The estimated variances of coefficient estimates are computed using the summary statistics we have already obtained. These will also be computed automatically linear regression fitting methods, as shown in the next section.

2.6 COMPARING MODELS: THE ANALYSIS OF VARIANCE

Computing the analysis of variance and F test by hand requires only the value of *RSS* and of $SSreg = SYY - RSS$. We can then follow the outline given in ALR[2.6].

SAS There are several SAS procedures that can do simple linear regression. The most basic procedure is `proc reg`, which prints at minimum the summary of the fit and the *ANOVA* table. For the Forbes data,

```
proc reg data = alr3.forbes;
 model Lpres = Temp;
 run;
```

is the smallest program for fitting regression. This will give the output shown in Table 2.3.

There are literally dozens of options you can add to the `model` statement that will modify output. For example, consider the following five statements:

```
 model Lpres=Temp;
```

Table 2.3 Output of `proc reg` with the `forbes` data.

```
                             The REG Procedure
                               Model: MODEL1
                         Dependent Variable: Lpres
                            Analysis of Variance
                             Sum of         Mean
Source             DF       Squares       Square    F Value    Pr > F

Model               1     425.63910    425.63910    2962.79    <.0001
Error              15       2.15493      0.14366
Corrected Total    16     427.79402

        Root MSE            0.37903    R-Square    0.9950
        Dependent Mean    139.60529    Adj R-Sq    0.9946
        Coeff Var           0.27150

                           Parameter Estimates
                   Parameter  Standard
Variable    DF     Estimate       Error    t Value  Pr > |t|

Intercept    1    -42.13778    3.34020     -12.62    <.0001
Temp         1      0.89549    0.01645      54.43    <.0001
```

```
model Lpres=Temp/noprint;
model Lpres=Temp/noint;
model Lpres=Temp/all;
model Lpres=Temp/CLB XPX I R;
```

All but the third will fit the same regression, but will differ on what is printed. The first will print the default output in Table 2.3. The second will produce no printed output, and is useful if followed by an `output` statement to save regression summaries for some other purpose. The third will fit a mean function with no intercept. The fourth will produce pages and pages of output, of everything the program knows about; you probably don't want to use this option very often (or ever). The last will print the default output confidence intervals for each of the regression coefficients, the matrix $X'X$, its inverse $(X'X)^{-1}$, and a printed version of the residuals. See documentation for `proc reg` for a complete list of options.

Remember that transformations must be done in a data step, not in `proc reg`.

proc glm Another important SAS procedure for fitting linear models is `proc glm`, which stands for the general linear model. `proc glm` is is more general than `proc reg` because it allows *class variables*, which is the name that SAS uses for factors. Also, `proc glm` has several different approaches to fitting *ANOVA* tables. The approach to *ANOVA* discussed in ALR[3.5] is *sequential* ANOVA, called Type I in SAS. The Wald or t-tests discussed in ALR are equivalent to SAS Type II, with no factors present. SAS Type III is not recommended in any regression problem. Table 2.4 gives a comparison of `proc reg` and `proc glm`.

2.7 THE COEFFICIENT OF DETERMINATION, R^2

2.8 CONFIDENCE INTERVALS AND TESTS

Confidence intervals and tests can be computed using the formulas in ALR[2.8], in much the same way as the previous computations were done.

SAS The SAS procedure `proc reg` will give confidence intervals for parameter estimates by adding the option `clb` to the `model` statement, so, for example,

```
 model Lpres=Temp/clb alpha=.01;
```

will print 99% confidence intervals; the default is 95% intervals.

A function called `tinv(p,df)` can be used in a data step to calculate quantiles for t-distribution with *df* degrees of freedom. Similar functions include `betainv(p,a,b)` for Beta distribution, `cinv(p,df)` for χ^2 distribution, `gaminv(p,a)` for Gamma distribution, `finv(p,ndf,ddf)` for F-distribution, and `probit(p)` for standard normal distribution. All of these functions can be used in the data step or any SAS procedure.

Prediction and fitted values

SAS It is easy in SAS to get fitted values and associated confidence intervals for each observation by adding the option `cli` keyword to the `model` statement in `proc reg`:

```
proc reg data=alr3.forbes alpha=.05;
  model Lpres=Temp /cli;
run;
```

To get predictions and confidence intervals at unobserved values of the predictors, you need to append new data to the data set, with a missing indicator (a period) for the response. In the Forbes data, for example, to get predictions at *Temp* $= 210, 200$, use

```
data forbes2;
  input Temp;
cards;
210
220
;
data forbes2;
  set alr3.forbes forbes2;
proc reg data=forbes2 alpha=.05;
```

Table 2.4 Comparison of `proc reg` and `proc glm`.

Property	`proc reg`	`proc glm`
Terms		
Class variables (called factors in ALR)	Not allowed; use dummy variables instead	Allowed
Polynomial terms and interactions	Not allowed; define them in data step	Allowed
Other transformations	Use a data step	Use a data step
Random effects	Not allowed	Not recommended; use `proc mixed` instead.
Fitting		
OLS and WLS	Yes	Yes
No intercept	Allowed	Allowed
Prediction/Fitted values		
Prediction at new values	Add new values for the predictors to the data set with missing values, indicated by a ".", for the response.	Allowed
Miscellaneous		
Covariance of estimates	Yes	No; but can be computed from model output
ANOVA tables	Overall *ANOVA* only.	Overall and sequential *ANOVA* and other types of SS, really different orders of fitting
R^2	Yes	Yes
Add/delete variables and refit	Allowed	Not allowed
Submodel selection	Allowed	Not allowed
Model diagnostics		
Diagnostic statistics	Yes	Yes
Graphical diagnostics	Yes	No

```
  model Lpres=Temp /cli;
run;
```

Table 2.5 Predictions and prediction standard errors for the `forbes` data.

```
                             The REG Procedure
                               Model: MODEL1
                          Dependent Variable: Lpres

                             Output Statistics
        Dep Var   Predicted     Std Error
Obs       Lpres       Value  Mean Predict       95% CL Predict     Residual

  1    131.7900    132.0357        0.1667    131.1532   132.9183    -0.2457
  2    131.7900    131.8566        0.1695    130.9717   132.7416    -0.0666
  3    135.0200    135.0804        0.1239    134.2304   135.9304    -0.0604
  4    135.5500    135.5282        0.1186    134.6817   136.3747     0.0218
  5    136.4600    136.4237        0.1089    135.5831   137.2642     0.0363
  6    136.8300    136.8714        0.1048    136.0332   137.7096    -0.0414
  7    137.8200    137.7669        0.0979    136.9325   138.6013     0.0531
  8    138.0000    137.9460        0.0969    137.1122   138.7798     0.0540
  9    138.0600    138.2146        0.0954    137.3816   139.0477    -0.1546
 10    138.0400    138.1251        0.0959    137.2918   138.9584    -0.0851
 11    140.0400    140.1847        0.0925    139.3531   141.0163    -0.1447
 12    142.4400    141.0802        0.0958    140.2469   141.9135     1.3598
 13    145.4700    145.4681        0.1416    144.6057   146.3306   0.001856
 14    144.3400    144.6622        0.1307    143.8076   145.5168    -0.3222
 15    146.3000    146.5427        0.1571    145.6682   147.4173    -0.2427
 16    147.5400    147.6173        0.1735    146.7288   148.5059    -0.0773
 17    147.8000    147.8860        0.1777    146.9937   148.7783    -0.0860
 18           .    145.9159        0.1480    145.0486   146.7831          .
 19           .    154.8708        0.2951    153.8469   155.8947          .

              Sum of Residuals                -3.3378E-13
              Sum of Squared Residuals            2.15493
              Predicted Residual SS (PRESS)       2.52585
```

The new data for the `input` variable *Temp* are entered after the `cards` statement; we need to use the same variable name as in the `forbes` data set so that these new values will be appended to the column of our interest.

To save the predicted values and associated confidence intervals for later computation, you can use `output` statement in `proc reg`. Here the information is saved to a data set called `m1`:

```
data forbes2;
  input Temp;
cards;
210
220
;
data forbes2;
  set alr3.forbes forbes2;
```

```
proc reg data=forbes2 alpha=.05;
  model Lpres=Temp /cli noprint;
  output out=m1 predicted=prediction L95=Lower_limit U95=Upper_limit;
run;
```

2.9 THE RESIDUALS

SAS The SAS procedure `proc reg` computes the residuals for the linear model we fit. A plot of residuals versus fitted values can be obtained using `plot` statement in `proc reg`. The keywords are `residual.`, the ordinary residuals, and `predicted.`; don't forget the trailing periods. The `/nostat nomodel` option suppresses the model fit information on the graph.

```
proc reg data=alr3.forbes;
  model Lpres=Temp /noprint;
  output out=m1 residual=res predicted=pred;
  plot residual.*predicted. /nostat nomodel; *the residual plot;
 run;
proc print data=work.m1 ;
 run;
```

The `output` statement was used to save the residuals and fitted values in a data set called `m1`. We then used `proc print` to print `m1`, but you could use this data set in other graphs or for other purposes.

Problems

2.2.

SAS In order to do part 2.2.3, you need to save the fitted values from the two regression models to two data sets, then apply SAS graphical procedure to the combined data set.

For part 2.2.5, you also need to save the predicted values and prediction standard errors, then use a data step to combine these data and the Hooker's data.

For part 2.2.6, you first need to combine the `forbes` data and the Hooker's data, with the response in the `forbes` data to missing. Then you fit the regression model with this combined data set. Since SAS provides predictions for the data with missing response as well, the computation of z-scores are still straightforward in this part.

2.7.

SAS A linear regression model without an intercept can be fit in SAS by adding `noint` option to the `model` statement.

2.10.

SAS The `where` *`conditions`*`;` statement can be used to select the cases of interest.

3 Multiple Regression

3.1 ADDING A TERM TO A SIMPLE LINEAR REGRESSION MODEL

SAS Using the `fuel2001` data, we first need to transform variables and create a new data set that includes them. To draw an added-variable plot in SAS, we use `proc reg` twice to get the sets of residuals that are needed. Using the `fuel2001` data,

```
data fuel;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  Income=Income/1000;
  logMiles=log2(Miles);
goptions reset=all;
title 'Added Variable Plot for Tax';
proc reg data=fuel;
  model Fuel Tax=Dlic Income logMiles;
  output out=m1 residual=res1 res2;
run;
proc reg data=m1;
  model res1=res2;
  plot res1*res2;
run;
```

The first `proc reg` has a `model` statement with two responses on the left side of the equal sign. This means that two regression models will be fit, one for each response. The `output` specification assigns `m1` to be the name of the

Table 3.1 Summary statistics from `proc means` for the `fuel2001` data.

```
                         The MEANS Procedure

Variable    N          Mean       Std Dev       Minimum       Maximum
Tax        51    20.1549020     4.5447360     7.5000000    29.0000000
Dlic       51   903.6778311    72.8578005   700.1952729       1075.29
Income     51    28.4039020     4.4516372    20.9930000    40.6400000
logMiles   51    15.7451611     1.4867142    10.5830828    18.1982868
Fuel       51   613.1288080    88.9599980   317.4923972   842.7917524
```

output, and the names for the two sets of residuals are `res1` and `res2`. The added-variable plot is just the plot of *res1* versus *res2*. We did this using `plot` inside the next `proc reg`; in this way, the fitted line and summary statistics are automatically added to the plot.

A low-quality version of all the added-variable plots (produced in the output window rather than a graphics window) can be produced using the `partial` option:

```
data=fuel;
  model Fuel = Tax Dlic Income logMiles/partial;
run;
```

A synonym for an added-variable plot is a *partial regression plot*.

3.2 THE MULTIPLE LINEAR REGRESSION MODEL

3.3 TERMS AND PREDICTORS

SAS The standard summary statistics in SAS can be retrieved using `proc means`, and the output is in Table 3.1.

```
data fuel;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  Income=Income/1000;
  logMiles=log2(Miles);
proc means data=fuel;
  var Tax Dlic Income logMiles Fuel;
run;
```

We will use `proc iml`, the interactive matrix language that we used in the last chapter, for several calculations and covariances. First, we get the sample correlations, starting with the fuel data set that we have just created.

```
proc iml;
use fuel;
```

```
read all var {Tax Dlic Income logMiles Fuel} into X;
size=nrow(X); one=j(1,size,1);
ave=one*X/size;
Xave=one`*ave;
W=X-Xave; Y=W`*W;
Xcov=Y/(size-1);
Xstd=sqrt(diag(Xcov)); Xstd_inv=inv(Xstd);
Xcor=Xstd_inv*Xcov*Xstd_inv;
print Xcor Xcov;
quit;
```

We have used `inv` to get the inverse of a matrix; it is equivalent to `X``. See output in Table 3.2.

Table 3.2 Sample correlation matrix and sample covariance matrix from `proc iml` for the `fuel2001` data.

```
      XCOR
         1 -0.085844 -0.010685 -0.043737 -0.259447
 -0.085844         1 -0.175961 0.0305907 0.4685063
 -0.010685 -0.175961         1 -0.295851 -0.464405
 -0.043737 0.0305907 -0.295851         1 0.4220323
 -0.259447 0.4685063 -0.464405 0.4220323         1

      XCOV
 20.654625  -28.4247 -0.216173 -0.295519 -104.8944
  -28.4247 5308.2591 -57.07045 3.3135435 3036.5905
 -0.216173 -57.07045 19.817074 -1.958037 -183.9126
 -0.295519 3.3135435 -1.958037 2.2103191 55.817191
 -104.8944 3036.5905 -183.9126 55.817191 7913.8812
```

3.4 ORDINARY LEAST SQUARES

SAS Continuing with using `proc iml` to do matrix calculations, the OLS estimates can be computed from $(X'X)^{-1}X'Y$; results are shown in Table 3.3.

```
data fuel;
  set alr3.fuel;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  Income=Income/1000;
  logMiles=log2(Miles);
  one=1;
proc iml;
use fuel2001;
read all var {one Tax Dlic Income logMiles} where (one^=. &
     Tax^=. & Dlic^=. & Income^=. & logMiles^=. & fuel^=.) into X;
read all var 'fuel' where (one^=. &
     Tax^=. & Dlic^=. & Income^=. & logMiles^=. & fuel^=.) into Y;
beta_hat=inv(t(X)*X)*t(X)*Y;
print beta_hat;
quit;
```

Table 3.3 OLS using `proc iml` for the `fuel2001` data.

```
         BETA_HAT
        154.19284
        -4.227983
        0.4718712
        -6.135331
        18.545275
```

The OLS estimates can be obtained using `proc reg` or `proc glm`. The output from `proc glm` is given in Table 3.4. The syntax is very similar to that for simple linear regression.

```
proc glm data=fuel;
  model Fuel=Tax Dlic Income logMiles;
run;
```

The only difference between fitting simple and multiple regression is in the specification of the model. All terms in the mean function appear to the right of the equal sign, separated by white space. The output for using `proc reg` would be similar, but would not include the extensive *ANOVA* tables shown.

3.5 THE ANALYSIS OF VARIANCE

SAS We recommend using `proc glm` for getting sequential analysis variance tables. For example,

```
proc glm data=fuel;
  model Fuel=Tax Dlic Income logMiles/ss1;
run;
```

produces the output shown in Table 3.4. This output gives the overall *ANOVA*, and by the sequential *ANOVA* discussed in ALR, and called *Type I ANOVA* by SAS. By using the option `/ss1`, we have suppressed SAS's default behavior of printing what it calls "Type III" sums of squares; these are not recommended (Nelder, 1977).

The order of fitting is determined by the order of the terms in the `model` statement. Thus

```
proc glm data=fuel;
  model Fuel=logMiles Income Tax Dlic  /ss1;
run;
```

would fit *logMiles* first, then *Income*, *Tax* and finally *Dlic*.

Table 3.4 Anova for nested models in `proc reg` for the `fuel2001` data.

```
The GLM Procedure

Dependent Variable: Fuel

                                  Sum of
Source                  DF       Squares   Mean Square  F Value  Pr > F
Model                    4   201994.0473    50498.5118    11.99  <.0001
Error                   46   193700.0152     4210.8699
Corrected Total         50   395694.0624

R-Square     Coeff Var      Root MSE     Fuel Mean
0.510480      10.58362      64.89122      613.1288

Source                  DF     Type I SS   Mean Square  F Value  Pr > F

Tax                      1   26635.27578   26635.27578     6.33  0.0155
Dlic                     1   79377.52778   79377.52778    18.85  <.0001
Income                   1   61408.17553   61408.17553    14.58  0.0004
logMiles                 1   34573.06818   34573.06818     8.21  0.0063

                                Standard
Parameter         Estimate         Error    t Value    Pr > |t|

Intercept      154.1928446   194.9061606       0.79      0.4329
Tax             -4.2279832     2.0301211      -2.08      0.0429
Dlic             0.4718712     0.1285134       3.67      0.0006
Income          -6.1353310     2.1936336      -2.80      0.0075
logMiles        18.5452745     6.4721745       2.87      0.0063
```

3.6 PREDICTIONS AND FITTED VALUES

SAS As with simple regression, we get predictions and prediction intervals by appending new data points to the data set with missing values for the response. See output in Table 3.5.

```
data fuel2;
  input Tax Dlic Income logMiles;
cards;
20 909 16.3 626
35 943 16.8 667
;
data fuel2; set fuel fuel2;
proc reg data=fuel2 alpha=.05;
  model Fuel=Tax Dlic Income logMiles /cli;
run;
```

We created a data file called `fuel2` consisting of two cases with values for four variables. We then redefined fule2 by appending it to the bottom of `fuel`; all

Table 3.5 Predictions and prediction standard errors at new values from `proc reg` for the `fuel2001` data.

```
                         The REG Procedure
                           Model: MODEL1
                     Dependent Variable: Fuel
... ... (omitted)
                        Output Statistics

      Dep Var  Predicted     Std Error
Obs      Fuel      Value  Mean Predict     95% CL Predict     Residual
  1  690.2644   727.2652       20.2801   590.4156  864.1147   -37.0007
  2  514.2792   677.4242       32.8388   531.0318  823.8166  -163.1450
... ... (omitted)
 50  581.7937   593.1223       18.5974   457.2446  729.0000   -11.3286
 51  842.7918   659.2930       18.7829   523.3120  795.2740   183.4988
 52         .      12008          3942       4072     19944          .
 53         .      12718          4209       4244     21192          .
... ... (omitted)
```

other variables are missing for these two cases. We then fit the regression, and obtain predictors for the two added cases.

Problems

4 Drawing Conclusions

The first three sections of this chapter do not introduce any new computational methods; everything you need is based on what has been covered in previous chapters. The last two sections, on missing data and on computationally intensive methods introduce new computing issues.

4.1 UNDERSTANDING PARAMETER ESTIMATES

4.1.1 Rate of change

4.1.2 Sign of estimates

4.1.3 Interpretation depends on other terms in the mean function

4.1.4 Rank deficient and over-parameterized models

SAS SAS provides several diagnostics when you try to fit a mean function in which some terms are linear combinations of others. Table 4.1 provides the output that parallels the discussion in ALR[4.1.4].

```
data BGS;
  set alr3.BGSgirls;
  DW9=WT9-WT2;
  DW18=WT18-WT9;
proc reg data=BGS;
  model Soma=WT2 DW9 DW18 WT9 WT18;
run;
```

Table 4.1 Output from an over-parameterized model for the `BGSgirls` data.

```
The REG Procedure
Model: MODEL1
Dependent Variable: SOMA

Number of Observations Read         70
Number of Observations Used         70

                        Analysis of Variance

                                Sum of          Mean
Source                  DF      Squares       Square  F Value  Pr > F


Model                    3     25.35982      8.45327    28.67  <.0001
Error                   66     19.45803      0.29482
Corrected Total         69     44.81786


Root MSE              0.54297    R-Square     0.5658
Dependent Mean        4.77857    Adj R-Sq     0.5461
Coeff Var            11.36264

NOTE: Model is not full rank. Least-squares solutions for the
      parameters are not unique. Some statistics will be misleading.
      A reported DF of 0 or B means that the estimate is biased.
NOTE: The following parameters have been set to 0, since the
      variables are a linear combination of other variables as shown.

 WT9 =  WT2 + DW9
WT18 =  WT2 + DW9 + DW18
                        Parameter Estimates

                     Parameter       Standard
Variable     DF       Estimate          Error    t Value    Pr > |t|

Intercept     1        1.59210        0.67425       2.36      0.0212
WT2           B       -0.01106        0.05194      -0.21      0.8321
DW9           B        0.10459        0.01570       6.66      <.0001
DW18          B        0.04834        0.01060       4.56      <.0001
WT9           0              0              .          .           .
WT18          0              0              .          .           .
```

Besides providing the correct *ANOVA* and other estimates, SAS correctly reports the linear combinations of the terms in the mean function. It then gives the estimates of parameters in the manner suggested in the text, by deleting additional terms that are exact linear combinations of previous terms in the mean function. SAS reports "B" degrees of freedom for the terms fit that are part of the exact linear combinations to indicate that the fit would have been different had the terms been entered in a different order.

SAS uses the term *model* for the *mean function*; in ALR, a model consists of the mean function and any other assumptions required in a problem.

If `proc glm` had been used in place of `proc reg`, the resulting output would have been similar, although the exact text of the diagnostic warnings is different.

4.2 EXPERIMENTATION VERSUS OBSERVATION

4.3 SAMPLING FROM A NORMAL POPULATION

4.4 MORE ON R^2

4.5 MISSING DATA

The data files that are included with ALR use "NA" as a place holder for missing values. Some packages may not recognize this as a missing value indicator, and so you may need to change this character using an editor to the appropriate character for your program.

SAS SAS can detect missing values with strings like 'NA', '.', '?', or any other that is not compatible with the rest values on the same variable(s). You do not need to tell SAS to skip those cases when you fit models, since it will do it automatically. As with most other programs, SAS will use cases that do not have missing values on any variables you use in a particular procedure.

```
data sleep;
  set alr3.sleep1;
  logSWS=log(SWS);
  logBodyWt=log(BodyWt);
  logLife=log(Life);
  logGP=log(GP);
proc reg data=sleep;
  model logSWS=logBodyWt logLife logGP;
run;
proc reg data=sleep;
  model logSWS=logBodyWt logGP;
  var logLife;
run;
```

The way you ask SAS to fit models with a particular subset of cases is by using the `var` statement. In the example above, the cases used will correspond to all cases observed on log(*Life*), even though that term is not in the model. You can guarantee that all models are fit to the same cases by putting all terms of interest in the `var` statement.

The SAS procedures `proc mi` and `proc mianalyze` provide tools for working with missing data beyond the scope of ALR; see Rubin (1987). Introduction to and examples on this procedure can be found at `support.sas.com/rnd/app/da/new/dami.html`.

4.6 COMPUTATIONALLY INTENSIVE METHODS

SAS SAS does not have predefined procedures for the bootstrap or other simulation methods, so you need use a *macro* for this purpose. The macro below is included in the SAS script file for this chapter. Macro writing can be much more complicated than most of the other SAS programming we discuss in this primer, so many readers may wish to skip this section. In our example, we use the `transact` data, which has $n = 261$ observations, and will produce a bootstrap similar to ALR[4.6.1]. Here is the macro, with some gentle annotation:

```
%let n=261;    transact has 261 rows;
%macro boots(B=1, seed=1234, outputdata=temp);  start macro def.;
%do i=1 %to &B;   repeat B times;
  data m1;   data step to create data file 'm1';
   rownum=int(ranuni(&seed*&i)*&n)+1;  select row number at random;
   set alr3.transact point=rownum;  add row 'rownum' to data set 'm1';
   j+1;    increment row counter;
   if j>&n then STOP; stop with sample size is n;
  still in the loop, use proc reg to fit with bootstrap sample;
     proc reg data=m1 noprint outest=outests (keep=INTERCEPT T1 T2);
        model Time=T1 T2;
     run;
 keep coef. estimates and then use proc append to save them;
     proc append base=&outputdata data=outests; run;
%end;  end the do loop;
%mend boots;  end the macro
  Do bootstrap B=999 times, and save as 'bootout';
%boots(B=999, seed=2004, outputdata=bootout);
```

If you want to use this macro with a different data set, you must: (1) change the definition of n; (2) substitute a different name for the data file, and (3) change the model and term names to the correct ones for your data set. Running the macro with $B = 999$ takes several minutes on a fast Windows computer.

The output from the last line is a new data set called `work.supp4boot1` with $B = 999$ rows. The ith row is the estimates of the three parameters from the ith bootstrap replication; the macro does no summarization. You can summarize the data using any of several tools. The simple program

```
proc univariate data=work.bootout;
histogram;
run;
```

will produce histograms for each predictor separately, along with a prodigious amount of output that includes the percentiles needed to get the bootstrap confidence intervals. Alternatively, you can select Solutions → Analysis → Interactive data analysis, and from the resulting window select the data set `bootout` from the `work` library. You can then select Analyze → Distribution to get a nice summary of the bootstraps.

We also provide in the script file for this chapter SAS programs that reproduce the summaries given in ALR. These are relatively obscure, and so we skip them here.

The simulation in ALR[4.6.3] employs a different idea, which is to add normal random noise to both the predictor and the response. The standard deviations for added random noise are obtained from data itself, namely, *SECPUE* and *SEdens*. The only arguments you can change in the macro below are the number of bootstrap B, the random seed `seed` and the output data set `outputdata`. They all have default values. All the bootstrap estimates of the parameter value are saved to the `outputdata`, whose name can be changed as an argument of the macro. In the output window, the point estimate and a 95% bootstrap confidence interval are displayed.

```
%macro boots2(B=1, seed=1234, outputdata=temp2);
%do i=1 %to &B;
data analysis;
  set alr3.npdata;
  rerr1=rannor(&seed*&i)*SECPUE;
  rerr2=rannor(&seed*&i*(&i+21))*SEdens;
Y=CPUE+rerr1;
X=Density+rerr2;
proc reg data=analysis noprint outest=outests (keep=X);
model Y=X /noint;
run;
proc append base=&outputdata data=outests; run;
%end;
proc univariate data=&outputdata noprint;
  output out=boot2stat mean=beta_mean
         pctlpts=2.5, 97.5 pctlpre=beta;
run;
proc print data=boot2stat; run;
%mend boots2;

***example as in the book, with B=999 bootstrap estimates***;
%boots2(B=999, seed=2004, outputdata=supp4boot2);
```

With B=999 and `seed`=2004, the results are:

```
Obs   beta_mean   beta2_5   beta97_5
  1     0.30745   0.23519    0.40662
```

5

Weights, Lack of Fit, and More

5.1 WEIGHTED LEAST SQUARES

SAS The `weights` option in `proc reg` or in `proc glm` is used for WLS.

```
data phy;
  set alr3.physics;
  w=1/SD**2;
proc reg data=phy;
  model y=x;
  weight w;
run;
```

Since the weights are the inverses of the *SD*s, we need to compute them in a `data` step before calling `proc reg`.

Prediction intervals and standard errors with WLS depend on the weight assigned to the future value. This means that the standard error of prediction is $(\hat{\sigma}^2/w_* + \text{sefit}(y|X = \mathbf{x}_*)^2)^{1/2}$, where w_* is the weight assigned to the future observation. The following SAS code obtains prediction interval for x_*=.1 and w_*=.04, and it uses the append-data technique we have used previously. The `cli` option on the `model` statement will generate prediction intervals.

```
data phy2;
  input x w;
  cards;
.1 .04
;
data phy2; set phy phy2;
proc reg data=phy2;
```

```
  model y=x /cli;
  weight w;
run;
```

In Table 5.1 the `Std Error Mean Predict` is sefit. The 95% t-confidence intervals for prediction correctly used sepred.

Table 5.1 Predictions and prediction confidence intervals for the `physics` data.

```
                              The REG Procedure
                                Model: MODEL1
                             Dependent Variable: y

                               Output Statistics

     Weight  Dep Var Predicted    Std Error
Obs Variable       y     Value Mean Predict    95% CL Predict    Residual
  1 0.003460 367.0000  331.6115       9.7215  262.9116  400.3113   35.3885
  2   0.0123 311.0000  300.8230       7.2080  262.6362  339.0098   10.1770
  3   0.0123 295.0000  281.7129       5.7614  244.8555  318.5704   13.2871
  4   0.0204 268.0000  267.9112       4.8259  238.9482  296.8742    0.0888
  5   0.0204 253.0000  258.3562       4.2688  229.8621  286.8502   -5.3562
  6   0.0278 239.0000  247.2086       3.7634  222.7009  271.7164   -8.2086
  7   0.0278 220.0000  233.9377       3.4542  209.6733  258.2021  -13.9377
  8   0.0278 213.0000  218.5435       3.5893  194.1751  242.9120   -5.5435
  9   0.0400 193.0000  193.0634       4.7764  171.0153  215.1116   -0.0634
 10   0.0400 192.0000  180.3234       5.6291  157.2300  203.4167   11.6766
 11        0        .  201.5568       4.2832  180.0542  223.0593         .

         Sum of Residuals                            0
         Sum of Squared Residuals             21.95265
         Predicted Residual SS (PRESS)        41.05411

NOTE: The above statistics use observation weights or frequencies.
```

5.1.1 Applications of weighted least squares

SAS To run polynomial regression, create polynomial terms in the data step, like squared terms and cubic terms, and then use `proc reg` to fit model with the original variables as well as these higher-order terms. Alternatively, you can use `proc glm` and specify the polynomials in the model statement. For a polynomial of degree two:

```
proc glm data=phy;
  model y=x x*x/ss1;
  weight w;
run; quit;
```

The *ANOVA* tables with Type I SS, the sequential sum of squares described in ALR are shown in Table 5.2.

Table 5.2 WLS quadratic regression model with `proc glm` for the `physics` data.

```
                       The GLM Procedure
                 Number of observations    10
                    Dependent Variable: y
                          Weight: w
                           Sum of
Source           DF       Squares  Mean Square  F Value   Pr > F
Model             2   360.7184868  180.3592434   391.41   <.0001
Error             7     3.2255311    0.4607902
Corrected Total   9   363.9440179

          R-Square     Coeff Var      Root MSE        y Mean
          0.991137      0.295019      0.678815      230.0922

Source    DF     Type I SS   Mean Square  F Value   Pr > F
x          1   341.9913694   341.9913694   742.18   <.0001
x*x        1    18.7271174    18.7271174    40.64   0.0004

                          Standard
Parameter    Estimate        Error  t Value  Pr > |t|
Intercept  183.830465    6.4590630    28.46    <.0001
x            0.970902   85.3687565     0.01    0.9912
x*x       1597.504726  250.5868546     6.38    0.0004
```

To do prediction using this polynomial model, we append the new data to the original data set, fit the regression model and obtain the predictions.

```
data phy2;
  input x w;
cards;
.1 .04
;
data phy2; set phy phy2;
proc glm data=phy2;
  model y=x x*x /predicted;
  weight w;
  output out=m2 predicted=ypred2;
run; quit;
```

5.1.2 Additional comments

5.2 TESTING FOR LACK OF FIT, VARIANCE KNOWN

SAS A SAS program to reproduce ALR[F5.1] is included in the script file for this chapter. The program is rather lengthy and complicated.

Calculations for the test for lack of fit are easily done using output from either `proc glm` or `proc reg`. To get significance levels, you need to be able to compute a p-value using a χ^2 distribution, and for this you can use the `chiprob` function in a `data` step. For example,

```
data qt;
  pval = 1-probchi(30,25);
  critval = cinv(.95,25);
  output;
proc print data=qt; run;
```

returns the output

```
Obs      pval      critval
 1     0.22429    37.6525
```

The first value is p-value based on the $\chi^2(25)$ distribution when the value of the test is 30, while the second is the critical value for the $\chi^2(25)$ distribution at level $\alpha = 1 - .05 = 0.05$. Similar functions are available for virtually all standard distributions. For example, to get a two-tailed p-value from the $t(30)$ distribution, if the value of the statistic is x, use the statement `2*(1-probt(abs(x),30))`.

5.3 TESTING FOR LACK OF FIT, VARIANCE UNKNOWN

SAS If we have one predictor x, we make use of the fact that *the sum of squares for lack of fit is the residual sum of squares for the regression of the response on x treated as a factor*. Thus, if we fit a model first with x and then with x as a factor, the (sequential) F-test for the factor will be F-test for lack of fit:

```
data temp;
  input x y;
  x2=x;
cards;
1 2.55
1 2.75
1 2.57
2 2.40
3 4.19
3 4.70
4 3.81
4 4.87
4 2.93
4 4.52
;
run;
proc glm data=temp;
  class x2;
  model y=x x2/ss1;
run; quit;
```

The SAS output in Table 5.3 shows that the p-value is about 0.39, providing no evidence for lack of fit. In a mean function with several terms, finding the

Table 5.3 Lack of fit test result with `proc glm`.

```
The GLM Procedure

Dependent Variable: y
                                     Sum of
Source                     DF       Squares   Mean Square  F Value  Pr > F
Model                       3    6.42749833    2.14249944     5.45  0.0378
Error                       6    2.35839167    0.39306528
Corrected Total             9    8.78589000


Source                     DF     Type I SS   Mean Square  F Value  Pr > F
x                           1    4.56925025    4.56925025    11.62  0.0143
x2                          2    1.85824808    0.92912404     2.36  0.1750
```

sum of squares for lack of fit is more of a challenge, and is omitted here. Code for this purpose is provided with the R and S-Plus languages.

5.4 GENERAL F TESTING

SAS SAS has a very general language using either `proc reg` or `proc glm` for performing specific F-tests. Using the fuel consumption data, `proc reg` can be used with the `test` statement:

```
data fuel;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  Income=Income/1000;
  logMiles=log2(Miles);
proc reg data=fuel;
  model Fuel=Tax Dlic Income logMiles;
  test Dlic=Income=0;
  test Dlic=Income=Tax=0;
run;
```

The first test statement gives the F-test

$$\begin{array}{ll}\text{NH:} & \mathrm{E}(\mathit{Fuel}|X) = \beta_0 + \beta_1 \mathit{Tax} + \beta_3 \log(\mathit{Miles}) \\ \text{AH:} & \mathrm{E}(\mathit{Fuel}|X) = \beta_0 + \beta_1 \mathit{Tax} + \beta_2 \mathit{Dlic} + \beta_3 \log(\mathit{Miles}) + \beta_4 \mathit{Income}\end{array}$$

while the second test is of

$$\begin{array}{ll}\text{NH:} & \mathrm{E}(\mathit{Fuel}|X) = \beta_0 + \beta_3 \log(\mathit{Miles}) \\ \text{AH:} & \mathrm{E}(\mathit{Fuel}|X) = \beta_0 + \beta_1 \mathit{Tax} + \beta_2 \mathit{Dlic} + \beta_3 \log(\mathit{Miles}) + \beta_4 \mathit{Income}\end{array}$$

The output for these tests is shown in Table 5.4. `proc glm` also has a `test` statement, but the syntax is different and the tests are more general. If your

Table 5.4 Anova for nested models with `proc reg` for the `fuel2001` data.

```
   Test 1 Results for Dependent Variable Fuel
                        Mean
Source        DF      Square   F Value   Pr > F
Numerator      2       54246     12.88   <.0001
Denominator   46  4210.86989

   Test 2 Results for Dependent Variable Fuel
                        Mean
Source        DF      Square   F Value   Pr > F
Numerator      3       43839     10.41   <.0001
Denominator   46  4210.86989
```

mean function includes factors (or class variables), then the usual tests of main effects and interactions will be produced by the program. For this purpose we once again suggest using *Type II tests*, so, for example,

```
proc glm data=fuel;
  model Fuel=Tax Dlic Income logMiles/e e2;
run;
```

will give an *ANOVA* table that has the correct F-tests for each term adjusted for each other term in the model.

5.5 JOINT CONFIDENCE REGIONS

SAS The code below will reproduce the confidence ellipse in ALR[F5.3]. It is included here for completeness, but is sufficiently complex that it is not of much use for data analysis, and so this may be skipped.

```
proc reg data=alr3.UN2 alpha=.05 outest=m1 tableout covout;
  model logFertility=logPPgdp Purban /noprint;
run;
proc iml;
use m1;
read all var {_RMSE_ logPPgdp Purban} into X;
sigmahat2=X[1,1]**2; *extract the variance estimate;
X=X[,2:3];
beta1=X[1,1]; *extract the coefficient estimate;
beta2=X[1,2]; *extract the coefficient estimate;
se=X[2,];
cov=X[8:9,]; *extract the covariance;
invcov=inv(cov); *the inverse of the covariance matrix;
z_val=probit(.975)*{1 -1};
int1=beta1+z_val*se[1];
int2=beta2+z_val*se[2];
one=j(1,2,1);
```

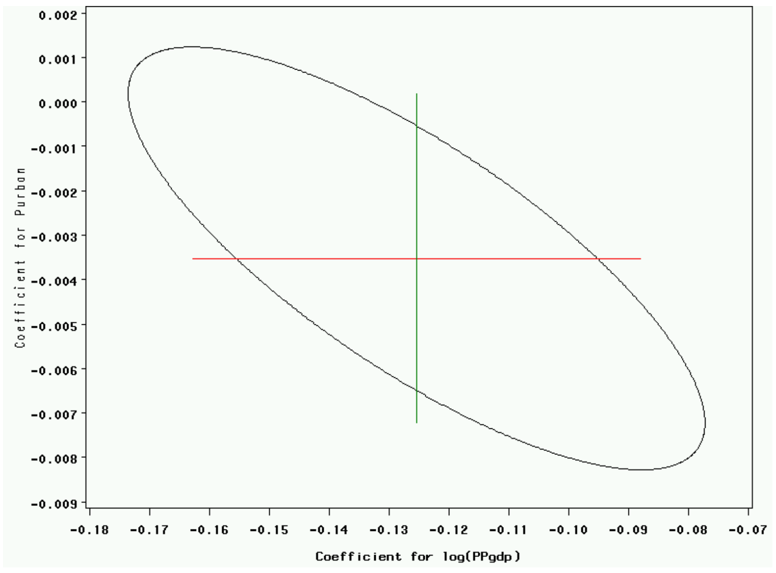


Fig. 5.1 95% confidence region for the `UN2` data.

```
f_val=2*finv(.95,2,49); *the cutoff value for the ellipse;
call eigen(V,E,invcov);
t=((1:1000)-1)#atan(1)#8/(1000-1);
a1=cos(t)*sqrt(f_val/V[1]);
a2=sin(t)*sqrt(f_val/V[2]);
invE=inv(E);
b1=invE[1,1]*a1+invE[1,2]*a2+beta1;
b2=invE[2,1]*a1+invE[2,2]*a2+beta2;
beta1=beta1*one;
beta2=beta2*one;
create cr var{b1 b2 int1 int2 beta1 beta2};
append var{b1 b2 int1 int2 beta1 beta2};
close cr;
quit;
goptions reset=all;
symbol v=point i=join l=1;
axis1 label=('Coefficient for log(PPgdp)');
axis2 label=(a=90 'Coefficient for Purban');
proc gplot data=cr;
  plot b2*b1 beta2*int1 int2*beta1
       /overlay haxis=axis1 hminor=0 vaxis=axis2 vminor=0;
run; quit;
```

To find the two main axes for the ellipse, the above code uses eigenvalue decomposition macro inside SAS: `eigen(V E matrix)`, and the keyword `call` has to be used before the macro `eigen`. `V` is used to save eigenvalues, `E` is used to save eigenvectors, both appear before the you want to decompose. Before we save the output, namely, the points on the ellipse, the confidence limits for each of the two variables, we multiply a two-vector of one's to the beta estimates. This is simply for drawing one-dimensional confidence intervals later on. Without doing this, SAS will only draw a single point for you, namely, only one endpoint of the interval, due to the way the output from `proc iml` is saved. If you are not convinced, try it again without the multiplication and you will see what is going to happen. The program could be converted to a SAS macro.

Problems

5.3.

The bootstrap used in this problem is different from the bootstrap discussed in ALR[4.6] because rather than resampling *cases* we are resampling *residuals*. Here is the general outline of the method:

1. Fit the model of interest to the data. In this case, the model is just the simple linear regression of the response y on the predictor x. Compute the test statistic of interest, given by ALR[E5.23]. Save the fitted values $\hat{y}$ and the residuals $\hat{e}$.

2. A bootstrap sample will consist of the original x and a new y^*, where $y^* = \hat{y} + e^*$. The ith element of e^* is obtained by sampling from $\hat{e}$ with replacement.

3. Given the bootstrap data (x, y^*), compute and save ALR[E5.23].

4. Repeat steps 2 and 3 B times, and summarize results.

6 Polynomials and Factors

6.1 POLYNOMIAL REGRESSION

SAS The following program reproduces ALR[F6.2]. We add uniform random noise to the data to get jittering.

```
data jitter;
  set alr3.cakes;
  X1=X1+.2*(ranuni(-1)-0.5); *add some noise to data;
  X2=X2+.5*(ranuni(-1)-0.5);
goptions reset=all;
symbol v=circle h=1;
proc gplot data=jitter;
  plot X2*X1 /hminor=0 vminor=0;
run; quit;
```

The uniform random number generator `ranuni(`*`seed`*`)` is used, with -1 indicating a system-chosen random seed. You can also change the multipliers, the values 0.2 and 0.5 used in the program, to control the magnitude of jittering.

There are at least two options for doing polynomial regression. One is to use `proc reg` with polynomial terms defined in the data step, or to use `proc glm`, where polynomial terms can be specified in `model` statement directly. For example, `x*x` is a quadratic term. Improved computational accuracy can be obtained using orthogonal polynomial regression in `proc orthoreg`; we will not further discuss this latter option.

6.1.1 Polynomials with several predictors

SAS Polynomial regression with multiple predictors can be fit with `proc reg` with pre-defined higher-order terms, or using `proc glm`, which allows the definition of higher-order terms in the `model` statement. The following uses `proc glm` to fit a full quadratic model. See output in Table 6.1.

```
proc glm data=alr3.cakes;
  model Y=X1 X1*X1 X2 X2*X2 X1*X2/ss1;
run; quit;
```

Table 6.1 Full quadratic model by `proc glm` for the `cakes` data.

```
                                  Sum of
Source             DF        Squares   Mean Square   F Value    Pr > F
Model               5    27.20482661    5.44096532     29.60    <.0001
Error               8     1.47074482    0.18384310
Corrected Total    13    28.67557143

        R-Square     Coeff Var      Root MSE        Y Mean
        0.948711      6.100376      0.428769      7.028571

Source       DF      Type I SS    Mean Square    F Value    Pr > F
X1            1     4.32315980     4.32315980      23.52    0.0013
X1*X1         1     2.13083104     2.13083104      11.59    0.0093
X2            1     7.43319538     7.43319538      40.43    0.0002
X2*X2         1    10.54541539    10.54541539      57.36    <.0001
X1*X2         1     2.77222500     2.77222500      15.08    0.0047

                                  Standard
Parameter        Estimate            Error    t Value    Pr > |t|
Intercept    -2204.484988      241.5807402      -9.13      <.0001
X1              25.917558        4.6589114       5.56      0.0005
X1*X1           -0.156875        0.0394457      -3.98      0.0041
X2               9.918267        1.1665592       8.50      <.0001
X2*X2           -0.011950        0.0015778      -7.57      <.0001
X1*X2           -0.041625        0.0107192      -3.88      0.0047
```

To draw ALR[F6.3A], we again use `proc glm` to generate the predicted values, then use `proc gplot` to plot all the points on one graph. Once again, the program for this plot is very long, and uses `proc iml`, the interactive matrix language.

```
proc iml; *create new data;
x1=j(3,50,.); x2=j(1,150,.);
x1[1,]=32+(0:49)*(38-32)/49; x2[1:50]=j(1,50,340);
x1[2,]=32+(0:49)*(38-32)/49; x2[51:100]=j(1,50,350);
x1[3,]=32+(0:49)*(38-32)/49; x2[101:150]=j(1,50,360);
create newdata var {x1 x2};
append var {x1 x2};
close newdata; quit;
data cakes; set newdata alr3.cakes;
```

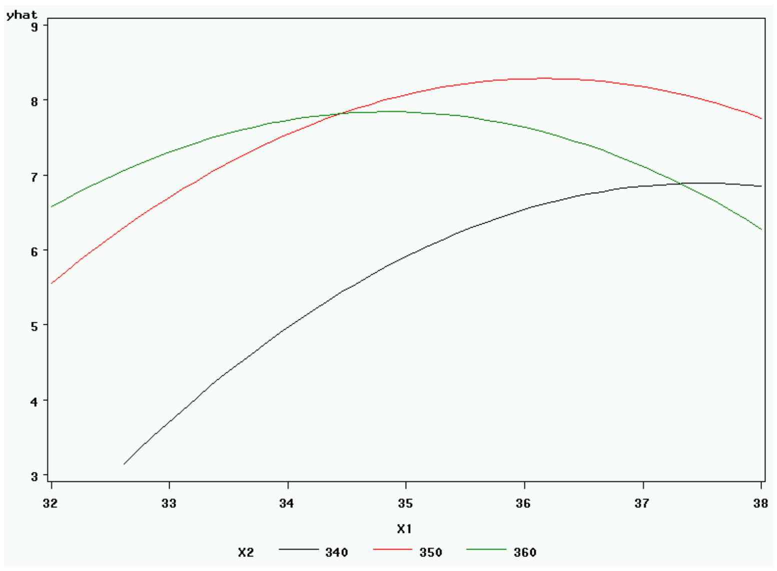


Fig. 6.1 Estimated response curve (ALR[F6.3A]).

```
proc glm data=cakes;
  model Y=X1 X1*X1 X2 X2*X2 X1*X2 /predicted;
  output out=m1 predicted=yhat; *save the predicted values;
run; quit;
symbol v=point i=join l=1;
proc gplot data=m1; where Y eq .; *choose the new data only;
  plot yhat*x1=x2
       /haxis=(32 to 38 by 1) hminor=0
        vaxis=(3 to 9 by 1) vminor=0;
run; quit;
```

To get estimated response curves, we need to get the predicted values. Thus we first generate these new predictor values, then obtain the prediction via `predicted` option in `proc glm`. Then we draw three estimated response curves from the output data set `m1`. The command `j(3,50,.)` gives a 3 by 50 matrix of ".", the missing value code, and the command `(0:49)` gives a sequence of integers from 0 to 49.

6.1.2 Using the delta method to estimate a minimum or a maximum

SAS The delta method is used to compute approximate standard errors of functions of parameter estimates. The computations can be done with `proc iml` using the formulas given in ALR. In this example, the `cakes` data is used. The mean function is $\mathrm{E}(y|X) = \beta_0 + \beta_1 x_2 + \beta_2 x_2^2$, and the nonlinear function of interest is $x_m = -\beta_1/(2\beta_2)$. As in the text, we ignore X_1 in this computation.

Use a data step to get a new data set
```
 data cakes;
  set alr3.cakes;
  X2sq=X2**2;
```
Use proc reg to get the covariance matrix of the estimates
```
proc reg data=cakes outest=m1 covout;
  model Y=X2 X2sq;
run;
```
Start proc iml, and do the computation
```
proc iml; *compute se based on delta method;
use m1;
read all var X2 X2sq into X;
b=X[1,];
cov=X[3:4,];
est=-b[1]/(2*b[2]);
db1=-1/(2*b[2]);
db2=b[1]/(2*b[2]**2);
se=sqrt(cov[1,1]*db1*db1+2*cov[1,2]*db1*db2+cov[2,2]*db2*db2);
print "-b1/(2*b2)", est [format=10.4];
print "StdErr (based on delta method)", se [format=10.4];
quit;
```

giving the results:

```
       -b1/(2*b2)
           EST
         354.2029

StdErr (based on delta method)
           SE
           2.0893
```

`est` is the estimate of x_m, `se` is the approximate standard error of it.

The procedure `proc nlmixed` uses the delta method to estimate nonlinear combinations of coefficients and their standard errors, using the `estimate` statement. For the `cakes` data, however, numerical problems arise, so we need to *center and scale* X_2 before fitting. We set $Z_2 = (X_2 - 350)/10$, and then fit the mean function

$$\begin{aligned} \mathrm{E}(Y|Z_2) &= \gamma_0 + \gamma_1 Z_2 + \gamma_2 Z_2^2 \\ &= \gamma_0 + \gamma_1 \left(\frac{X_2 - 350}{10}\right) + \gamma_2 \left(\frac{X_2 - 350}{10}\right)^2 \end{aligned}$$

Differentiating this last equation and setting the result to zero, we find that this this parameterization the maximum will occur at $x_m = 350 - 5\gamma_1/\gamma_2$. Here is the SAS code:

```
data cakes;
  set alr3.cakes;
  Z2 = (X2-350)/10;
```

```
proc nlmixed data = cakes;
  parms gam0-gam2=0 s2=1;
  yhat=gam0+gam1*Z2+gam2*Z2*Z2;
  model Y~normal(yhat,s2);
  estimate '350-5*gam1/gam2' 350-5*gam1/gam2;
run; quit;
```

which gives the following (edited) output:

```
Parameter Estimates
                      Standard
  Parameter  Estimate    Error    DF  t Value  Pr > |t|
  gam0         7.6838   0.3075    14    24.99    <.0001
  gam1         0.9639   0.3201    14     3.01    0.0093
  gam2        -1.1467   0.3322    14    -3.45    0.0039
  s2           0.8196   0.3098    14     2.65    0.0192

Additional Estimates
                            Standard
Label             Estimate     Error     DF       Lower      Upper
350-5*gam1/gam2     354.20    1.8519     14      350.23     358.17
```

The point estimate agrees with the value computed with `proc iml`, but the standard error is smaller. The discrepancy is caused by computation of the estimate of variance. When using `proc iml`, we used the estimate of σ^2 computed in the usual way as the *RSS* divided by its df, which is 11 for this problem. `proc nlmixed` estimates σ^2, by *RSS* divided by n, which for this problem is 14. If we multiply the standard error by $\sqrt{14/11}$, we get 2.0892, essentially agreeing with the solution computed with `proc iml`. The lesson to be learned is that different programs do different things, and you need to know what your program is doing.

6.1.3 Fractional polynomials

6.2 FACTORS

Factors are a slippery topic because different computer programs will handle them in different ways. In particular, while SAS and SPSS use the same default for defining factors, JMP, R and S-Plus all used different defaults. A factor represents a qualitative variable with say a levels by $a-1$ (or, if no intercept is in the model, possibly a) dummy variables. ALR[E6.16] describes one method for defining the dummy variables, using the following rules:

1. If a factor A has a levels, create a dummy variables $U_1, \ldots, U_a$, such that U_j has the value one when the level of A is j, and value zero everywhere else.

2. Obtain a set of $a-1$ dummy variables to represent factor A by dropping one of the dummy variables. For example, using the default coding in

Table 6.2 How `proc glm` creates dummies.

Data		Design Matrix						
				A			B	
A	B	mu	A1	A2	A3	B1	B2	B3
1	1	1	1	0	0	1	0	0
1	2	1	1	0	0	0	1	0
2	3	1	0	1	0	0	0	1
3	1	1	0	0	1	1	0	0
3	2	1	0	0	1	0	1	0
3	3	1	0	0	1	0	0	1

R, the first dummy variable U_1 is dropped, while in SAS and SPSS the last dummy variable is dropped.

3. JMP and S-Plus use a completely different method.

Most of the discussion in ALR assumes the R default for defining dummy variables.

SAS One way to work with factors is to create your own dummy variables and work with them in place of the factors. For some users, this may make computations and results clearer. When using `proc reg`, there is no choice because this procedure does not recognize factors. Dummy variables can be created with `if` statements in a `data` step. For example, `if D=1 then D1=1; else D1=0; if D=2 then D2=1; else D2=0;` creates two dummy variables. The dummies can be used to fit regression model as factors in `proc reg`.

With `proc glm`, you can use `class` variables, which treats all the variables listed in that statement as factors, so you can fit model with those factors directly. Think of a class variable with m levels as a collection of m dummy variables, one for each level. In fitting one of the dummies is usually redundant, and SAS will drop the one corresponding to the last level. The default order of the levels is their alphabetical order; this order can be controlled with the `order=`*`option`* in the `proc glm` statement. Table 6.2 shows the default class variable parameterization in `proc glm`. If you fit a mean function using `model Y = A B;`, from Table 6.2 the columns for `A3` and `B3` would not be used in fitting.

6.2.1 No other predictors

SAS To obtain ALR[T6.1A], which is part of Table 6.3, one can use the following SAS code:

```
proc glm data=alr3.sleep1;
  class D;
  model TS=D /ss1 noint solution clparm;
 run;
```

We used the `ss1` option to get only Type I sums of squares; the *noint* option to fit with no intercept, the `solution` option to print estimates, and the `clparm` option to print confidence intervals for the estimates.

Table 6.3 Complete output from `proc glm` with only one factor for the `sleep1` data.

```
The GLM Procedure

Dependent Variable: TS
                                         Sum of
Source                     DF           Squares    Mean Square  F Value  Pr > F
Model                       5       6891.717825    1378.343565    97.09  <.0001
Error                      53        752.412175      14.196456
Uncorrected Total          58       7644.130000

R-Square     Coeff Var      Root MSE       TS Mean
0.378001      35.77238      3.767818      10.53276

Source                     DF       Type I SS    Mean Square  F Value  Pr > F
D                           5     6891.717825    1378.343565    97.09  <.0001

                                         Standard
Parameter           Estimate                Error    t Value    Pr > |t|

D         1      13.08333333       0.88808333      14.73      <.0001
D         2      11.75000000       1.00699185      11.67      <.0001
D         3      10.31000000       1.19148882       8.65      <.0001
D         4       8.81111111       1.25593949       7.02      <.0001
D         5       4.07142857       1.42410153       2.86      0.0061

Parameter         95% Confidence Limits

D         1      11.30206374  14.86460292
D         2       9.73023014  13.76976986
D         3       7.92017607  12.69982393
D         4       6.29201550  11.33020672
D         5       1.21504264   6.9278145
```

ALR[T6.1B] uses the default parameterization for R, which drops the first level by default. If you fit in SAS without the `noint` option, you will get output similar to ALR[T6.1B], except the "baseline" level will be the last level, not the first level. You could get exactly the same estimates as in ALR[T6.1B] using the following:

```
data s2;
 set alr3.sleep1;
 D2 = D;
 if D=1 then D2=6;
 output;
prog glm data=s2;
 class D2;
 model TS=D2/solution;
run;
```

We have reordered the factor D so its first level is alphabetically the last level, and it becomes the baseline as in the text.

6.2.2 Adding a predictor: Comparing regression lines

SAS Suppose Y is the generic name of the response variable, X the generic name of the continuous predictor, and D is a SAS class variable. The four models described in ALR[6.2.2] can be fit using `proc glm` using the following four `model` statements:

`model Y = X D X*D;`
 `model Y = D X*D/noint;`
	Model 1, most general

`model Y = X D;`
 `model Y = X D/noint;`
	Model 2, parallel lines

`model Y = X D*X;`
 `model Y = D*X;`
	Model 3, common intercept

`model Y = X;`	Model 4, coincident lines

The different choices for the model statements will use different parameters to specify the same mean function. For example, assuming D has 3 levels, and D_1, D_2 and D_3, are, respectively, dummy variables for levels $1, 2, 3$ of D, `model Y = X D X*D;` will specify

$$\mathrm{E}(Y|X) = \beta_0 + \beta_1 X + \beta_2 D_1 + \beta_3 D_2 + \beta_4 D_1 X + \beta_5 D_2 X$$

while `model Y = D X*D/noint;` specifies

$$\mathrm{E}(Y|X) = \eta_1 D_1 + \eta_2 D_2 + \eta_3 D_3 + \eta_4 D_1 X + \eta_5 D_2 X + \eta_6 D_3 X$$

Both of these will have the same residual sum of squares, and that is all that is needed for the F-tests.

For the sleep data, the following four models can be fit:

```
data sleep;
  set alr3.sleep1;
  logBodyWt=log2(BodyWt);
proc glm data=sleep; *general model;
  class D;
  model TS=D D*logBodyWt /noint;
run; quit;
proc glm data=sleep; *parallel model;
  class D;
  model TS=D logBodyWt /noint;
run; quit;
proc glm data=sleep; *common intercept;
```

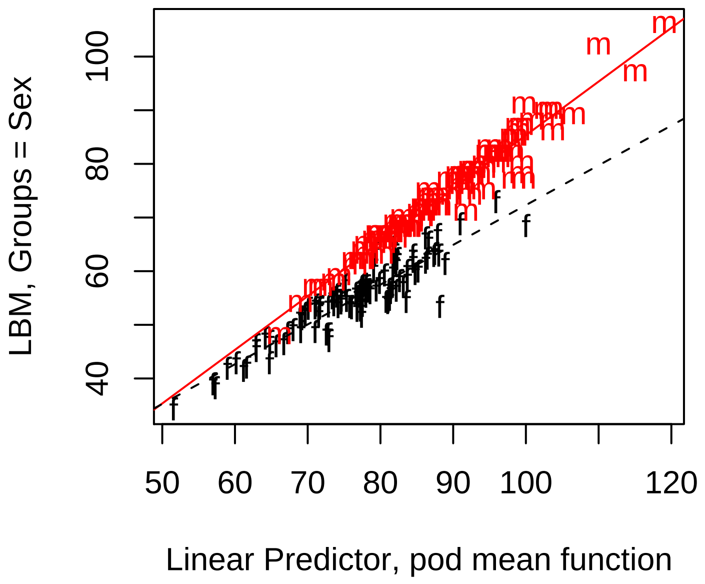


Fig. 6.2 ALR[F6.8] in color.

```
  class D;
  model TS=D*logBodyWt;
run; quit;
proc glm data=sleep; *coincident lines;
  class D;
  model TS=logBodyWt;
run; quit;
```

Given the residual sums of squares from these models, computing the F-test by hand is probably easiest.

6.3 MANY FACTORS

6.4 PARTIAL ONE-DIMENSIONAL MEAN FUNCTIONS

ALR[F6.8] is much more compelling in color, and is shown here as Figure 6.2.

SAS Since POD models are really nonlinear regression models, they can be fit in SAS using the general procedure `proc nlmixed`. This procedure is sufficiently general that we can use POD models with other types of errors, such as binomial errors or Poisson errors; see Cook and Weisberg (2004). The output is given by Table 6.4 with the iteration history omitted.

```
 Call proc nlmixed to fit the POD model
 proc nlmixed data=alr3.ais;
   Starting values for parameters are required, and you may need better ones
  parms beta0=1 beta1=1 beta2=1 beta3=1 beta4=1 eta1=1 sig2=1;
  g=beta2*Ht+beta3*Wt+beta4*RCC;
  LBM_exp=beta0+beta1*Sex+g+eta1*Sex*g;
  model LBM~normal(LBM_exp,sig2);
  predict g out=m1;
  predict LBM_exp out=m2;
run; quit;
```

Table 6.4 Edited output of the POD model with `proc nlmixed` for the `ais` data.

```
                         Parameter Estimates
              Standard
Param Estimate   Error   DF t Value Pr > |t| Alpha    Lower    Upper  Gradient
beta0 -14.6563  6.3486  202   -2.31   0.0220  0.05 -27.1744  -2.1382  0.000016
beta1  12.8472  3.6889  202    3.48   0.0006  0.05   5.5735  20.1208   0.00001
beta2   0.1463 0.03376  202    4.33   <.0001  0.05  0.07969   0.2128  0.002007
beta3   0.7093 0.02380  202   29.81   <.0001  0.05   0.6624   0.7563  0.000578
beta4   0.7248  0.5768  202    1.26   0.2104  0.05  -0.4126   1.8621  0.000051
eta1   -0.2587 0.03402  202   -7.61   <.0001  0.05  -0.3258  -0.1917  0.001157
sig2    5.8708  0.5842  202   10.05   <.0001  0.05   4.7190   7.0227  -0.00006
```

The following will plot separately for each group in the POD direction.

```
data m1; set m1; g=Pred; keep LBM g Sex;
data pod; set m1; set m2; LBM_exp=Pred; keep LBM g LBM_exp Sex;
proc sort data=pod; by g;
goptions reset=all;
symbol1 v=point c=black i=join l=1;
symbol2 v=point c=red i=join l=2;
axis2 label=(a=90 'LBM');
axis3 label=(a=90 '');
proc gplot data=pod;
  plot LBM_exp*g=Sex /vaxis=axis2 hminor=0 vminor=0;
  plot2 LBM*g=Sex /vaxis=axis3 vminor=0 nolegend;
run; quit;
```

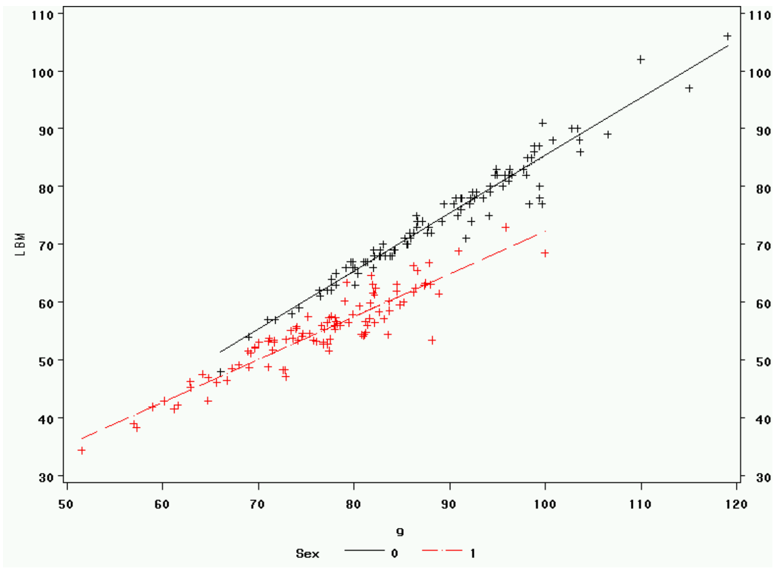


Fig. 6.3 Summary graph for the POD mean function for the `ais` data.

6.5 RANDOM COEFFICIENT MODELS

SAS Random coefficient models can be fit in SAS using `proc mixed`. We illustrate with the `chloride` data and the output is shown in Table 6.5 and Table 6.6.

```
proc mixed data=alr3.chloride method=reml covtest;
  class Marsh Type;
  model Cl=Type Month /noint cl;
  random INTERCEPT /type=un subject=Marsh;
run; quit;
proc mixed data=alr3.chloride method=reml covtest;
  class Marsh Type;
  model Cl=Type Month /noint cl;
  random Type /type=un subject=Marsh;
run; quit;
```

Each of the two `mixed` procedures gives parameter estimates (with confidence limits), estimated covariance matrix, and model fit statistics like $-2\times$log-likelihood.

`method=reml` specifies that the method to estimate the parameters is REML (*REstricted Maximum Likelihood*). `covtest` tells SAS to output the standard errors for covariance parameter estimates. Option `cl` gives Wald confidence intervals for parameter estimates (for fixed effects). The keyword `INTERCEPT` in the `random` statement refers to random intercept for each level of `subject=Marsh`. Similarly, `random Type /subject=Marsh` assumes random `Type`

Table 6.5 Random intercept for each level of *Marsh* with `proc mixed` for the `chloride` data.

```
                 The Mixed Procedure
               Convergence criteria met.
             Covariance Parameter Estimates
                               Standard       Z
Cov Parm    Subject   Estimate     Error   Value     Pr Z
UN(1,1)     Marsh       177.83    102.61    1.73   0.0415
Residual               40.7964   12.3276    3.31   0.0005

                    Fit Statistics
         -2 Res Log Likelihood           219.7
         AIC (smaller is better)         223.7
         AICC (smaller is better)        224.2
         BIC (smaller is better)         224.1

            Null Model Likelihood Ratio Test
            DF    Chi-Square      Pr > ChiSq
             1         23.61          <.0001

               Solution for Fixed Effects
                        Standard
Effect Type     Estimate    Error  DF t Value Pr > |t| Alpha    Lower   Upper
Type   Isolated  -5.5038   7.7172  22   -0.71   0.4832  0.05 -21.5084 10.5007
Type   Roadside  45.0680   7.0916  22    6.36   <.0001  0.05  30.3610 59.7751
Month             1.8538   0.5299  22    3.50   0.0020  0.05   0.7549  2.9527

              Type 3 Tests of Fixed Effects
                  Num     Den
        Effect      DF      DF    F Value    Pr > F
        Type         2      22      22.56    <.0001
        Month        1      22      12.24    0.0020
```

effect within each level of `Marsh`. Option `type=UN` specifies a *unstructured* covariance matrix. The default is `type=VC`, which refers to *variance components*.

SAS does not provide *ANOVA* comparison for nested models, and it does not provide confidence limits for covariance parameter estimates. The *ANOVA* model comparison is made by likelihood ratio test, which compares the difference in $-2\times$log-likelihood with a χ^2 distribution (the corresponding degrees of freedom is equal to the difference in number of parameters between the large model and the small model). The output gives the change in $-2\times$log-likelihood from 219.7 to 214.1, which equals 5.6, with two degrees of freedom. Hence p-value=`1-probchi(5.6,2)`$=$ 0.061. SAS gives p-values for covariance parameter estimates based on normal distribution, which is appropriate only in large samples.

Table 6.6 Random intercept and random *Type* effect for each level of *Marsh* with `proc mixed` for the `chloride` data.

```
                  The Mixed Procedure
                Convergence criteria met.
              Covariance Parameter Estimates
                                Standard       Z
Cov Parm    Subject   Estimate     Error   Value     Pr Z
UN(1,1)     Marsh       4.3219   12.2704    0.35   0.3623
UN(2,1)     Marsh            0         .       .        .
UN(2,2)     Marsh       312.87    230.95    1.35   0.0878
Residual               40.0770   11.9382    3.36   0.0004

                     Fit Statistics

         -2 Res Log Likelihood          214.1
         AIC (smaller is better)        222.1
         AICC (smaller is better)       223.8
         BIC (smaller is better)        222.9

            Null Model Likelihood Ratio Test
            DF    Chi-Square      Pr > ChiSq
             3         29.16          <.0001

                Solution for Fixed Effects
                         Standard
Effect Type      Estimate    Error  DF t Value Pr > |t| Alpha    Lower   Upper
Type   Isolated  -5.3342   3.9873   7   -1.34   0.2228  0.05 -14.7626  4.0942
Type   Roadside  45.1451   8.7550   7    5.16   0.0013  0.05  24.4429 65.8473
Month             1.8279   0.5171  22    3.53   0.0019  0.05   0.7555  2.9002

               Type 3 Tests of Fixed Effects
                   Num     Den
        Effect      DF      DF    F Value    Pr > F
        Type         2       7      18.65    0.0016
        Month        1      22      12.50    0.0019
```

7 Transformations

7.1 TRANSFORMATIONS AND SCATTERPLOTS

7.1.1 Power transformations

7.1.2 Transforming only the predictor variable

SAS Many of the methods described in ALR center on looking at a graph or a few graphs to select transformations. The graphs generally consist of both data and fitted curves, and these seem to be very tedious to produce. For example, ALR[F7.3] is a simple scatterplot with three different fitted curves added to it. The following program will draw the graph.

```
data ufcwc;
  set alr3.ufcwc;
  Dbh_tran1=(Dbh**(-1)-(-1))/(-1);
  Dbh_tran2=log(Dbh);
proc reg data=ufcwc;
  model Height=Dbh;
  output out=trans predicted=Hhat;
run;
proc reg data=ufcwc;
  model Height=Dbh_tran1;
  output out=trans1 predicted=Hhat1;
run;
proc reg data=ufcwc;
  model Height=Dbh_tran2;
  output out=trans2 predicted=Hhat2;
```

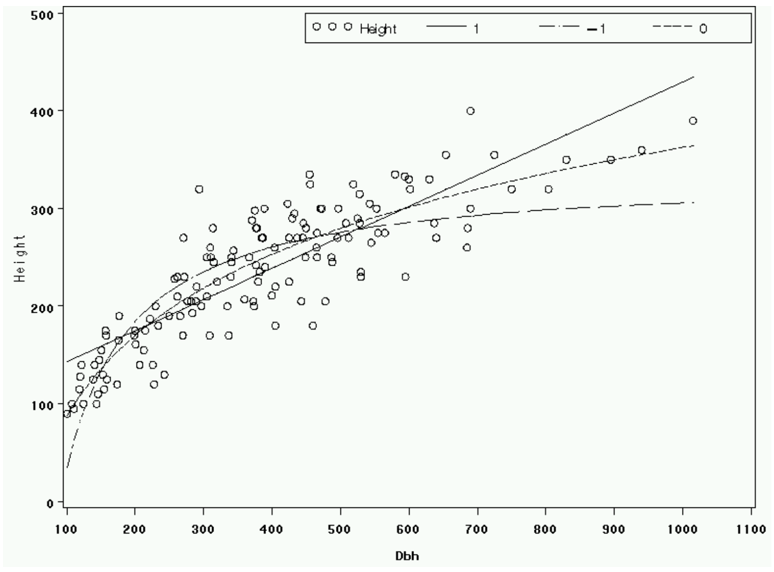


Fig. 7.1 *Heights* versus *Diameter* for the `ufcwc` data.

```
run;
data trans; set trans; set trans1; set trans2;
proc sort data=trans; by Dbh;
goptions reset=all;
symbol1 v=circle h=1 c=black;
symbol2 v=point h=1 c=black i=join l=1;
symbol3 v=point h=1 c=black i=join l=2;
symbol4 v=point h=1 c=black i=join l=3;
axis2 label=(a=90 'Height');
legend1 label=none value=(h=1 font=swiss 'Height' '1' '-1' '0')
  position=(top right inside) cborder=black;
proc gplot data=trans;
  plot Height*Dbh=1 Hhat*Dbh=2 Hhat1*Dbh=3 Hhat2*Dbh=4
       /overlay hminor=0 vaxis=axis2 vminor=0 legend=legend1;
run; quit;
```

From the three regressions, we keep the predicted values, called `Hat`, `Hat1` and `Hat2`, and then combine them into another data set with a data step. The remainder of the code sets up and draws the plot.

Rather than relying on visualization to select a transformation, we can use a numerical method. The idea is to use least squares to estimate the parameters in the mean function:

$$\mathrm{E}(Y|X=x) = \beta_0 + \beta_1 \psi_S(x, \lambda) \tag{7.1}$$

This will estimate β_0, β_1 and λ simultaneously. This can be done using non-linear least squares (see Chapter 11), with a program similar to the following:

```
proc nlin data=alr3.ufcwc;
  parms beta0 = 111 beta1=.3 lam=1;
  model Height = beta0 +
   beta1*( if(abs(lam) < .001) then log(Dbh) else (Dbh**lam -1)/lam);
  output out=nlinout predictor=yhat;
run;
```

Nonlinear least squares is fit using `proc nlin`. In this procedure, the user must specify *starting values* for the parameters, and for this we set `lam`= 1, and the values for `beta0` and `beta1` were obtained by first fitting the regression of *Dbh* on *Height* using `proc reg`. The model statement matches (7.1), substituting the definition of $\psi_S(x, \lambda)$. The output from this fit is, in part,

```
                              Approx
Parameter      Estimate    Std Error    Approximate 95% Conf. Limits
th0              -378.0        249.8       -872.0       116.0
th1             90.8127      79.5272     -66.4586       248.1
lam              0.0479       0.1522      -0.2531      0.3488
```

from which we estimate λ to be about 0.05, with an interval estimate of between about $-.25$ and $+.35$. Any transformation in is range is likely to be useful, and since $\lambda = 0$ is included, we would use the logarithmic transformation.

The following program will use the output from the last program to draw a graph of the data with the fitted line superimposed.

```
proc sort data=nlinout; by Dbh;
goptions reset=all;
symbol1 v=circle h=1 c=black;
symbol2 v=point h=1 c=black i=join l=1;
proc gplot data=nlinout;
 plot Height*Dbh=1 yhat*Dbh=2/overlay;
run; quit;
```

`proc sort` is used to order the data according to the values on the horizontal axis of the plot, and the remaining statements draw the plot.

7.1.3 Transforming the response only

SAS The inverse response plots like ALR[F7.2] are very inconvenient in SAS (although writing a macro that would automate this process would make an interesting and useful student project); a sample program that will give this plot is included in the script file for this chapter. SAS does have a procedure that implements the Box-Cox method, however.

7.1.4 The Box and Cox method

SAS The procedure `proc transreg` implements the Box-Cox procedure for selecting a response transformation; the other options in this procedure are not covered in ALR. Here is the syntax for getting Box-Cox transformations, using the highway data:

```
data highway;
  set alr3.highway;
  logLen=log2(Len); logADT=log2(ADT);
  logTrks=log2(Trks); logSigs1=log2((Len*Sigs+1)/Len);
proc transreg data=highway;
  model boxcox(Rate /convenient lambda=-1 to 1 by .005)=
        identity(logLen logADT logTrks Slim Shld logSigs1);
run;
```

Table 7.1 Box-Cox transformation for `Rate` in the `highway` data.

```
       The TRANSREG Procedure
     Transformation Information
          for BoxCox(Rate)
  Lambda      R-Square    Log Like

     ...          ...          ...
  -0.830         0.67    -0.57565
  -0.825         0.67    -0.54340
  -0.820         0.67    -0.51140 *
  -0.815         0.67    -0.47963 *
     ...          ...          ...
  -0.245         0.70     1.40755 *
  -0.240         0.70     1.40780 <
  -0.235         0.70     1.40774 *
     ...          ...          ...
  -0.010         0.70     1.09198 *
  -0.005         0.70     1.07794 *
   0.000 +       0.70     1.06359 *
   0.005         0.70     1.04893 *
   0.025         0.70     0.98725 *
     ...          ...          ...
   0.315         0.70    -0.45778 *
   0.320         0.70    -0.49165 *
   0.325         0.70    -0.52582
   0.330         0.70    -0.56028
     ...          ...          ...

< - Best Lambda
* - Confidence Interval
+ - Convenient Lambda
```

The required keyword `identity` specifies no further transformation of the predictors. In the example, zero for log transformation is a convenient power if it is in the confidence interval, labelled by the `+`. As usual, we have transformed predictors in the data step, although `proc transreg` does have some facility for this within the command. The response is specified by `boxcox(Rate)`. The keywords specify marking a convenient power, and also the search range for

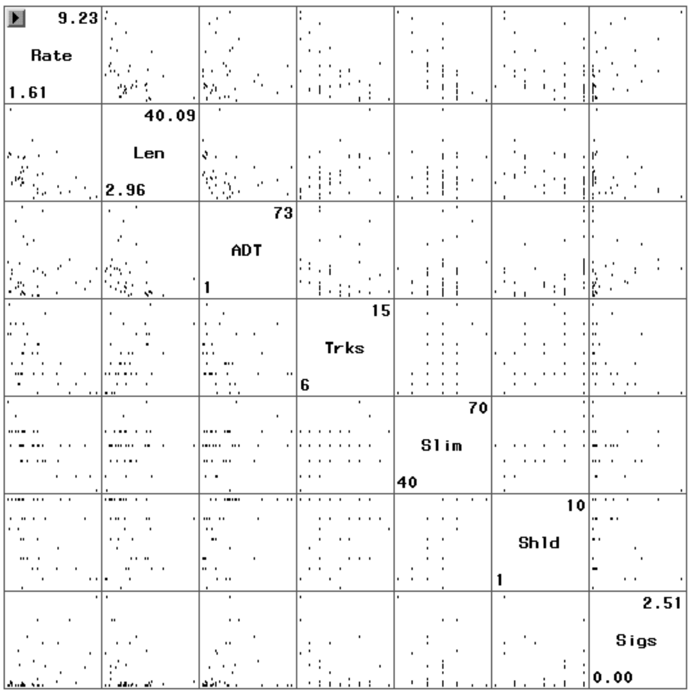


Fig. 7.2 Scatterplot matrix for the `highway` data.

λ, here between -1 and 1 by steps of .005. Table 7.1 shows the result using the Box-Cox method, where the "Lambda" denoted by a `<` is optimal, and the "Lambda" denoted by a `+` is a convenient choice.

7.2 TRANSFORMATIONS AND SCATTERPLOT MATRICES

The scatterplot matrix is the central graphical object in learning about regression models. You should draw them all the time; all problems with many continuous predictors should start with one.

SAS Use Solutions → Analysis → Interactive Data Analysis to get a scatterplot matrix. The equivalent `proc insight` code is

```
proc insight data=alr3.highway;
  scatter Rate Len ADT Trks Slim Shld Sigs
         *Rate Len ADT Trks Slim Shld Sigs;
run;
```

You must type the variable names twice so that you can get all the pairwise scatterplots. The symbol * connects the variables to be plotted on the vertical axis and the variables to be plotted on the horizontal axis. And there is no easy procedure for adding *loess* smooth curves and OLS regression lines to the scatterplot matrix.

7.2.1 The 1D estimation result and linearly related predictors

7.2.2 Automatic choice of transformation of the predictors

SAS No SAS tools for multivariate Box-Cox transformations are available, although once again this could be a reasonable student project to write macro for this purpose.

7.3 TRANSFORMING THE RESPONSE

SAS One can follow Section 7.1.4 on the Box-Cox method in SAS `proc transreg` to obtain the a Box-Cox transformation power for the response. An alternative is to do a grid search over an interval of interest. You need to define power transformations for the response in the data step, then fit multiple models for the transformed responses and pick the power that gives you the smallest RSS.

7.4 TRANSFORMATIONS OF NON-POSITIVE VARIABLES

SAS Using the Yeo-Johnson transformations in SAS requires writing your own code for the Box-Cox procedure. The example below shows how to compute the Yeo-Johnson transformation for a particular choice of λ, but not how to select the optimal value of λ.

```
data highway;
 set alr3.highway;
 *to get the Yeo-Johnson transformation power lambda=1.25;
 if (Rate ge 0) then Rate_YJ=((Rate+1)**1.25-1)/1.25;
   else Rate_YJ=-((1-Rate)**(2-1.25)-1)/(2-1.25);
```

8 Regression Diagnostics: Residuals

8.1 THE RESIDUALS

SAS Options to `proc glm` and `proc reg` are used to compute and save diagnostic quantities to a data file. Additionally, residuals and related quantities can be used to draw simple graphs inside these procedures. The following program

```
proc reg data=alr3.ufcwc;
 model Height=Dbh;
 output out=m1 predicted=pred residual=res h=hat press=pres;
run;
```

will fit the regression mean function specified, and create a new data set (`work.m1`) consisting of the original data, plus new variables called `pred` for the predicted values, `res` for the residuals, `hat` for the leverages, and `pres` for the *PRESS* residuals. Table 8.1 gives relevant diagnostic quantities that can be saved.

You can also use residuals and related statistics *within* the call to `proc reg` or `proc glm`. In particular

```
proc reg data=alr3.ufcwc;
 model Height=Dbh;
 plot residual.*predicted.;
 plot press.*Dbh;
run;
```

will produce two scatterplots, the first of residuals versus fitted values, and the second of *PRESS* versus *Dbh*. (To plot a keyword like `residual`, you

Table 8.1 Selected quantities with a value for every case that can be saved using the `output` statement in `proc reg` or `proc glm`.

SAS name	Quantity
residuals	ordinary residuals ALR[E8.4]
student	standardized residuals ALR[E9.3]
rstudent	Studentized residuals ALR[E9.4]
cookd	Cook's distance ALR[E9.6]
h	leverages ALR[E8.7]
press	*PRESS* residuals ALR[PROB8.4]
stdi	sepred for predicting new data at old X-values, ALR[E3.23]
stdp	sefit at old X-values ALR[E3.24]
predicted	fitted values $X\hat{\beta}$

must append a period to the end of the name.) A disadvantage to using this plotting mechanism is that you can't add smoothers or other enhancements.

Also,

```
proc reg data=alr3.ufcwc;
 model Height=Dbh;
 print all;
run;
```

will print lists of many diagnostic statistics. This should only be used with very small sample sizes!

8.1.1 Difference between $\hat{e}$ and e

8.1.2 The hat matrix

SAS To find the leverages h_{ii}, the diagonals of the hat matrix, using `proc reg`, type:

```
data ufcwc;
  set alr3.ufcwc;
proc reg data=ufcwc;
  model Height=Dbh;
  output out=m1 h=hat;
run;
```

This code *saves* the leverages, so they are available to other procedures for summarization or graphing.

8.1.3 Residuals and the hat matrix with weights

As pointed out in ALR[8.1.3], the residuals for WLS are $\sqrt{w_i} \times (y_i - \hat{y}_i)$. Whatever computer program you are using, you need to check to see how residuals are defined.

SAS SAS defines the residuals as in ALR[E8.4] to be *unweighted* residuals. *In* WLS *regression, these are not the correct residuals to use in diagnostic procedures.*[1] Other diagnostic statistics, such as leverages, ALR[E8.7], the standardized residuals, ALR[E9.3], called `STUDENT` in SAS, and the outlier test ALR[E9.2], called `RSTUDENT` in SAS, are correctly computed by SAS. We recommend that with WLS residual plots should use standardized or Studentized residuals, not raw residuals.

If you want to use the correct weighted (or Pearson) residuals ALR[E8.13], the following SAS code computes, prints, and plots them for the `physics` data.

```
data physics;
  set alr3.physics;
  w=1/sd**2;
proc reg data=physics;
  model y=x;
  weight w;
  output out=m1 residual=res predicted=yhat h=hat;
run;
data m1; set m1;
  pearson_res=sqrt(w)*res; *compute correct residuals;
proc print data=m1; run;
proc gplot data=m1;
  plot pearson_res*yhat=1 /vref=0 hminor=0 vaxis=axis2 vminor=0;
run;
quit;
```

8.1.4 The residuals when the model is correct

8.1.5 The residuals when the model is not correct

8.1.6 Fuel consumption data

SAS Solutions → Analysis → Interactive Data Analysis starts `proc insight`, which can be used to produce multiple residual plots. You can also start this procedure with the following program.

```
data fuel2001;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
```

[1]SAS also uses the unweighted residuals in `proc insight`.

```
  Income=Income/1000;
  logMiles=log2(Miles);
proc insight data=fuel2001;
  fit Fuel=Tax Dlic Income logMiles;
  scatter R_Fuel*Tax Dlic Income logMiles P_Fuel /label=State;
  scatter Fuel*P_Fuel /label=State;
run;
```

The `label` option allows user to specify a labelling variable. The variables `R_yname` for residuals and `P_yname` for fitted values are computed by the `fit` statement. These graphs are interactive, and you can use the mouse to identify the labels for individual cases.

8.2 TESTING FOR CURVATURE

SAS The following SAS code will reproduce ALR[T8.1], as shown in Table 8.2.

```
data fuel2001;
  set alr3.fuel2001;
  Dlic=Drivers*1000/Pop;
  Fuel=FuelC*1000/Pop;
  Income=Income/1000;
  logMiles=log2(Miles);
proc reg data=fuel2001; *this proc reg is simply to retrieve fitted values;
  model Fuel=Tax Dlic Income logMiles /noprint;
  output out=m1 predicted=pred;
run;
data m1; *this data step is to add quadratic terms to the old data set;
  set m1;
  Tax2=Tax**2; Dlic2=Dlic**2;
  Income2=Income**2; logMiles2=logMiles**2;
  pred2=pred**2;
proc reg data=m1; *fit models with one quadratic term added each time;
  model Fuel=Tax Tax2 Dlic Income logMiles;
  model Fuel=Tax Dlic Dlic2 Income logMiles;
  model Fuel=Tax Dlic Income Income2 logMiles;
  model Fuel=Tax Dlic Income logMiles logMiles2;
  model Fuel=Tax Dlic Income logMiles pred2;
  ods output ParameterEstimates=est;
  *save the parameter estimates and t-test results;
run;
data est; set est; Term=scan(Variable,1,'2');
data est; set est; where Term ne Variable;
                  *keep the test statistics for added terms only;
  if Term='pred' then do;
     Probt=2*probnorm(tValue); Term='Tukey test'; end;
proc print data=est; var Term tValue Probt; run;
```

Table 8.2 Tests for curvature for the `fuel2001` data.

Obs	Term	tValue	Probt
1	Tax	-1.08	0.2874
2	Dlic	-1.92	0.0610
3	Income	-0.08	0.9334
4	logMiles	-1.35	0.1846
5	Tukey test	-1.45	0.1482

8.3 NONCONSTANT VARIANCE

8.3.1 Variance Stabilizing Transformations

8.3.2 A diagnostic for nonconstant variance

SAS The score test for nonconstant variance can be computed in SAS using the `proc model` method; the score test is the same as the *Bruesch-Pagan test*. The use of `proc model` is relatively complex and is beyond the scope of this primer.

If you don't mind doing a side calculation by hand, you con compute the score test using the following program:

```
proc reg data=alr3.snowgeese;
  model photo=obs1;
  output out=m1 residual=res predicted=fit;
run;
data m2;
  set m1;
  u = res*res;
proc reg data=m2;
  model u=obs1;
run;
```

According to the discussion in ALR[8.3.3], if RSS_1 is the residual sum of squares from the first of the regressions above, and $SSreg_2$ is the regression sum of squares from the second regression, then the score test (for variance as a function of *obs1*) is $.5SSreg_2/(RSS_1/n)$, with df given by the df for *SSreg*, in the example equal to one. To get the score test as a function of fitted values, replace the second regression above by

```
proc reg data=m2;
  model u=fit;
run;
```

With only one term in the mean function beyond the intercept, the score test for variance proportional to the mean is the same as the score test for variance as a function of the single predictor, so both of the above tests will have the same value (81.414) for the test statistic.

The SAS data step can be used to provide statistical tables. For example, the score test described above has value 81.414 and 1 df. Although computation of the significance level for this test is unnecessary because the value of the statistic is so large, we illustrate the computation:

```
data temp;
 pvalue = 1-probchi(81.414,1);
proc print data=temp; run;
```

We provide in the scripts for this chapter a program that uses `proc iml` to get the score test without the side calculation.

8.3.3 Additional comments

8.4 GRAPHS FOR MODEL ASSESSMENT

8.4.1 Checking mean functions

SAS There is no automated procedure in SAS for producing model checking plots. You have to save all the fitted values, including OLS prediction and smoother, use `proc gplot` to display all the information. In the `ufcwc` data, for example, the mean function has only one predictor *Dbh* and response *Height*. Unless you write a macro, possibly based on the programs given below, this methodology is unusable in SAS.

We can reproduce ALR[F8.11] as follows:

```
proc loess data=alr3.ufcwc;
  model Height=Dbh /smooth=.6;
  ods output OutputStatistics=m1;
run;
proc reg data=ufcwc;
  model Height=Dbh;
  output out=m2 predicted=OLS_pred;
run;
data fit; set m1; set m2;
proc sort data=fit; by Dbh;
goptions reset=all;
symbol1 v=circle c=black;
symbol2 v=point c=black i=join l=33;
symbol3 v=point c=black i=join l=1;
axis2 label=(a=90 'Height');
proc gplot data=fit;
  plot DepVar*Dbh=1 OLS_pred*Dbh=2 Pred*Dbh=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run;
```

For a model with multiple predictors, the following SAS code generates mean checking plots ALR[F8.13] for the `UN2` data:

```
proc reg data=alr3.UN2;
```

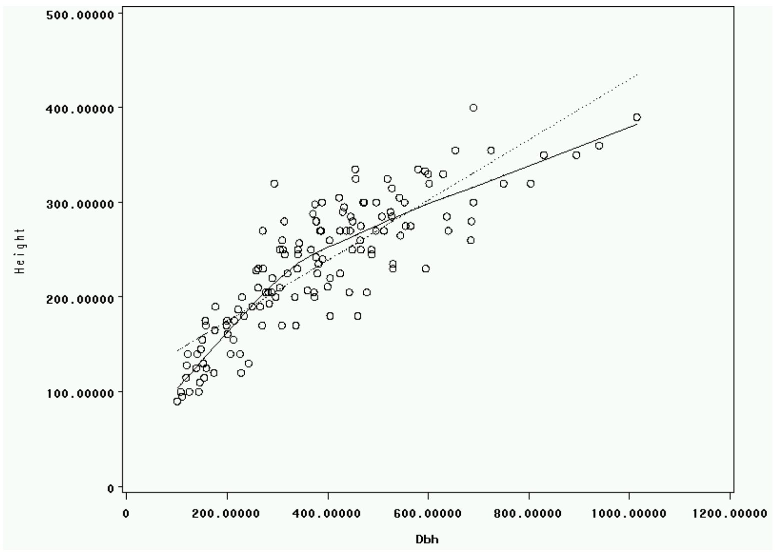


Fig. 8.1 SAS mean checking plot for the `ufcwc` data.

```
  model logFertility=logPPgdp Purban;
  output out=ols predicted=OLS_pred;
run;
goptions reset=all;
symbol1 v=circle h=1 c=black;
symbol2 v=point c=black i=join l=33;
symbol3 v=point c=black i=join l=1;
axis2 label=(a=90 'log(Fertility)');
*---------------------------------------------------;
*BEGIN plotting alr{F8.13A};
proc reg data=ols;
  model OLS_pred=logPPgdp;
  output out=mmp1 predicted=mmp1;
run;
proc loess data=ols;
  model logFertility=logPPgdp /smooth=.67;
  ods output OutputStatistics=loess1;
run;
data fit1; set mmp1; set loess1;
proc sort data=fit1;
  by logPPgdp; *sort by logPPgdp before plotting alr{F8.13A};
proc gplot data=fit1;
  plot DepVar*logPPgdp=1 mmp1*logPPgdp=2 Pred*logPPgdp=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run;
quit;
```

```
*END plotting alr{F8.13A};
*---------------------------------------------------;
*BEGIN plotting alr{F8.13B};
proc reg data=ols;
  model OLS_pred=Purban;
  output out=mmp2 predicted=mmp2;
run;
proc loess data=ols;
  model logFertility=Purban /smooth=.67;
  ods output OutputStatistics=loess2;
run;
data fit2; set mmp2; set loess2;
proc sort data=fit2;
  by Purban; *sort by Purban before plotting alr{F8.13B};
proc gplot data=fit2;
  plot DepVar*Purban=1 mmp2*Purban=2 Pred*Purban=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run;
quit;
*END plotting alr{F8.13B};
*---------------------------------------------------;
*BEGIN plotting alr{F8.13C};
data mmp3; set ols; mmp3=OLS_pred; FittedValues=OLS_pred;
proc loess data=ols;
  model logFertility=OLS_pred /smooth=.67;
  ods output OutputStatistics=loess3;
run;
data fit3; set mmp3; set loess3;
proc sort data=fit3;
  by FittedValues; *sort by FittedValues before plotting alr{F8.13C};
proc gplot data=fit3;
  plot DepVar*FittedValues=1 mmp3*FittedValues=2 Pred*FittedValues=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run;
quit;
*END plotting alr{F8.13C};
*---------------------------------------------------;
*BEGIN plotting alr{F8.13D};
data random;
  set ols; a1=ranuni(-1); a2=ranuni(-1);
  RandomDirection=a1*logPPgdp+a2*Purban;
proc reg data=random;
  model OLS_pred=RandomDirection;
  output out=mmp4 predicted=mmp4;
run;
proc loess data=random;
  model logFertility=RandomDirection /smooth=.67;
  ods output OutputStatistics=loess4;
run;
```

```
data fit4; set mmp4; set loess4;
proc sort data=fit4; by RandomDirection;
*sort by RandomDirection before plotting alr{F8.13D};
proc gplot data=fit4;
  plot DepVar*RandomDirection=1
       mmp4*RandomDirection=2
       Pred*RandomDirection=3
       /overlay hminor=0 vaxis=axis2 vminor=0;
run;
quit;
*END plotting alr{F8.13D};
```

The part of the SAS program generates a random combination of *logPPgdp* and *Purban* in the data step, where `a1` and `a2` are both uniform random numbers with a *random seed* -1. You can change the seed to any arbitrary value by the statement `seed=`*`value`*. Finally, the `RandomDirection` is used as predictor in `proc reg` and `proc loess`.

8.4.2 Checking variance functions

SAS Variance checking plots also require several steps, and are generated following ALR[A.5]. One important step in the SAS code below is `proc sql`, which appends a single-number data set to another larger data set. The general form of its `create table` statement is: `create table` *`newdata-name`* `as select` *`variable-list`* `from` *`olddata-name1`*`,` *`olddata-name2`* `<,`*`olddata-name3`*`,...>;`.

The * in the `create table` statement refers to *"all available variables in the selected data sets"*. What this procedure below does is to assign the single value (for example, `sigma` in `anova1`) to each row of the larger data set (for example, `ols1`). By doing this, we then can use the appended variable in later calculation. The following will reproduce ALR[F8.15A] and ALR[F8.15B] for the `UN2` data:

```
data UN2;
  set alr3.UN2;
  Purban2=Purban**2;
goptions reset=all;
symbol1 v=circle h=1 c=black;
symbol2 v=point c=black i=join l=33;
symbol3 v=point c=black i=join l=1;
axis1 label=('Fitted values, original model');
axis2 label=('Fitted values, quadratic term added');
axis3 label=(a=90 'log(Fertility)');
*--------------------------------------------------;
*BEGIN plotting alr{F8.15A};
proc reg data=UN2;
  model logFertility=logPPgdp Purban;
  output out=ols1 predicted=OLS_pred;
  ods output Anova=anova1;
run;
```

```
data anova1; set anova1; sigma=sqrt(MS); where Source='Error'; keep sigma;
proc sql; create table ols1 as select * from ols1, anova1; quit;
data mmp1; *compute OLS_mean+/-se;
  set ols1; FittedValues=OLS_pred;
  upper1=FittedValues+sigma; lower1=FittedValues-sigma;
proc print data=mmp1; run;
proc loess data=ols1;
  model logFertility=OLS_pred /smooth=.67 residual;
  ods output OutputStatistics=loess1;
run;
data loess1; set loess1; r2=Residual**2; mean_loess=Pred; y=DepVar;
proc loess data=loess1; *following alr{A.5};
  model r2=OLS_pred /smooth=.67;
  ods output OutputStatistics=loess_se1;
run;
data loess1; *compute loess+/-se;
  set loess1; set loess_se1;
  if Pred le 0 then do;
    upper_loess1=mean_loess; lower_loess1=mean_loess; end;
  else do;
  upper_loess1=mean_loess+sqrt(Pred);
  lower_loess1=mean_loess-sqrt(Pred); end;
data fit1; set mmp1; set loess1; *combine data sets to draw alr{F8.15A};
proc sort data=fit1;
  by FittedValues; *sort by FittedValues before plotting;
proc gplot data=fit1;
  plot y*FittedValues=1 (FittedValues upper1 lower1)*FittedValues=2
       (mean_loess upper_loess1 lower_loess1)*FittedValues=3
       /overlay haxis=axis1 hminor=0 vaxis=axis3 vminor=0;
run;
quit;
*END plotting alr{F8.15A};
*----------------------------------------------------;
*BEGIN plotting alr{F8.15B};
proc reg data=UN2;
  model logFertility=logPPgdp Purban Purban2;
  output out=ols2 predicted=OLS_pred;
  ods output Anova=anova2;
run;
data anova2; set anova2; sigma=sqrt(MS); where Source='Error'; keep sigma;
proc sql; create table ols2 as select * from ols2, anova2; quit;
data mmp2; *compute OLS_mean+/-se;
  set ols2; FittedValues=OLS_pred;
  upper2=FittedValues+sigma; lower2=FittedValues-sigma;
proc print data=mmp2; run;
proc loess data=ols2;
  model logFertility=OLS_pred /smooth=.67 residual;
  ods output OutputStatistics=loess2;
run;
```

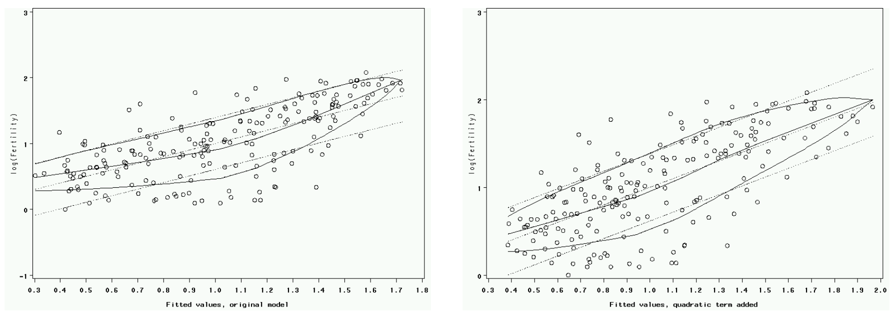


Fig. 8.2 Marginal model plots with standard deviation smooths as in ALR[F8.15].

```
data loess2; set loess2; r2=Residual**2; mean_loess=Pred; y=DepVar;
proc loess data=loess2; *following alr{A.5};
  model r2=OLS_pred /smooth=.67;
  ods output OutputStatistics=loess_se2;
run;
data loess2; *compute loess+/-se;
  set loess2; set loess_se2;
  if Pred le 0 then do;
    upper_loess2=mean_loess; lower_loess2=mean_loess; end;
  else do;
  upper_loess2=mean_loess+sqrt(Pred);
  lower_loess2=mean_loess-sqrt(Pred); end;
data fit2; set mmp2; set loess2; *combine data sets to draw alr{F8.15A};
proc sort data=fit2;
  by FittedValues; *sort by FittedValues before plotting;
proc gplot data=fit2;
  plot y*FittedValues=1 (FittedValues upper2 lower2)*FittedValues=2
       (mean_loess upper_loess2 lower_loess2)*FittedValues=3
       /overlay haxis=axis2 hminor=0 vaxis=axis3 vminor=0;
run;
quit;
*END plotting alr{F8.15B};
```

9 Outliers and Influence

9.1 OUTLIERS

9.1.1 An outlier test

SAS Both proc glm and proc reg can compute the studentized residuals rstudent that are used as the basis of the outlier test. Finding the maximum absolute residual, and then using the Bonferroni inequality as outlined in ALR, requires writing a complex SAS program, which we describe below.

9.1.2 Weighted least squares

9.1.3 Significance levels for the outlier test

SAS In the following example, an outlier test is performed, assuming that assumptions for outlier test in ALR[9.1.1] are met. This test uses the largest absolute Studentized residual. The df is also from the output from proc reg using the ods statement. The data we use here is UN2.

```
proc reg data=alr3.UN2;
  model logFertility=logPPgdp Purban;
  output out=m1 rstudent=rstu;
run;
data m1;
  set m1;
  retain casenum 0;
  casenum=casenum+1;
  absrstu=abs(rstu);
proc sort data=m1; by descending absrstu; run;
data m1; set m1 (obs=10); p=min(1,193*2*(1-probt(absrstu,190-1)));
proc print data=m1; run;
```

We first use `proc reg` to get the Studentized residuals, t_i. In the next `data` step, we create the absolute values $t_i|$ of `rstudent` and a variable `casenum` that gives the observation number. This latter variable can serve as an identifier in problems in which the points are not labelled. `proc sort` ordered the data according to the values of $|t_i|$. The next data step sets p equal to the Bonferroni p-values; *you must supply the sample size, $n = 193$ in the example, and the number of df for error, 190, in the example* suitable for your problem. The statistics for the 10 cases with the largest $|t_i|$ are then printed:

```
Obs   logPPgdp  logFertility  Purban  Locality        absrstu    p

  1   8.434628    0.33647224      41  Moldova         2.73895    1
  2   9.424166    0.13976194      67  Armenia         2.69971    1
  3   9.575539    0.13976194      68  Ukraine         2.63789    1
  4  10.663558    0.09531018      67  Bulgaria        2.38718    1
  5  10.125413    0.26236426      43  Bosnia-Herzeg   2.34340    1
  6   9.231221    0.33647224      57  Georgia         2.32188    1
  7  12.857398    1.60140574      77  Oman            2.30490    1
  8   9.157347     0.3435897      61  N.Korea         2.29487    1
  9  10.249113    0.18232156      70  Belarus         2.27244    1
 10   11.94398      1.773256      49  Equatorial.Gu   2.19654    1
```

None of the Studentized residuals are particularly large in this example, and all the Bonferroni-adjusted p-values are equal to one.

For problems that are not too large, you can identify the largest Studentized residual simply by examining a list of them. Suppose, for example, that $n = 100, p' = 5$, and the largest absolute Studentized residual has the value 3.83. You can compute the significance level for the outlier test from

```
data temp;
 pvalue = min(1,100*2*(1-probt(3.83,200-5-1)));
proc print data=temp; run;
```

Giving the following output:

```
Obs     pvalue
 1     0.017292
```

9.1.4 Additional comments

9.2 INFLUENCE OF CASES

SAS You can obtain various influence statistics from SAS output. For example, the following SAS code will give you raw residual, studentized residual, leverage and Cook's distance:

```
proc reg data=alr3.UN2;
```

```
  model logFertility=logPPgdp Purban;
  output out=m1 residual=res rstudent=rstu h=hat cookd=cookd;
run;
```

The `reweight` and `refit` options in `proc reg` provide a simple mechanism for refitting after deleting a few cases. In the following example, we fit the mean function $\log(\mathit{Fertility}) = \beta_0 + \beta_1 \log(\mathit{PPgdp}) + \beta_2 \mathit{Purban}$ using the `UN2` data. We first delete case 2 and refit. We then replace it, and remove cases 1, 3 and 6. Finally, we restore all the deleted cases.

```
data UN2;
  set alr3.UN2;
  retain casenum 0;
  casenum=casenum+1;
proc reg data=UN2;
  model logFertility=logPPgdp Purban;
run;
reweight casenum=2; refit; print; run; *delete case 2;
reweight undo; *restore the last deleted case;
reweight casenum=1;
reweight casenum=3;
reweight casenum=6;
refit; print; run; *delete case 1,3,6 from the data set;
reweight allobs /reset; *restore all previously deleted cases;
```

The key statements here are `reweight` *`condition(s)`* and `refit`. `reweight` changes the weights on the selected cases. You can specify the new (common) weight for the selected cases that satisfy certain *condition(s)*. We choose to omit the `weight=value` option because the default is to set the weights of those cases to *zero*. Another option for `reweight` statement is `reset`, for example, in the last `reweight` statement in the code above. What that statement does is to *reset* the weights of `allobs` (namely, *all observations*) to their initial values. The statement `reweight undo;` will *undo* the last `reweight` statement.

9.2.1 Cook's distance

SAS The plot of Cook's distance versus observation number using the `rat` data can be obtained as follows:

```
goptions reset=all;
proc reg data=rat;
  model y=BodyWt LiverWt Dose;
  output out=m1 cookd=cookd;
  plot cookd.*obs.;
run;
```

9.2.2 Magnitude of D_i

SAS We present in the script for this chapter a program to reproduce a version of ALR[F9.3].

9.2.3 Computing D_i

9.2.4 Other measures of influence

SAS Added variable plots are discussed in Chapter 3.

9.3 NORMALITY ASSUMPTION

SAS QQ plots can be drawn with the `plot` statement in `proc reg`.

```
data heights;
  infile "c:/My Documents/alr3/heights.txt" firstobs=2;
  input Mheight Dheight;
goptions reset=all;
*********************************;
**qqnorm plot for heights data***;
*********************************;
proc reg data=heights;
  model Dheight=Mheight;
  title 'QQ Plot';
  plot r.*nqq. /noline mse cframe=ligr;
run;
*********************************;
**qqnorm plot for transact data**;
*********************************;
data transact;
  infile "c:/My Documents/alr3/transact.txt" firstobs=2;
  input T1 T2 Time;
proc reg data=transact;
  model Time=T1 T2;
  title 'QQ Plot';
  plot r.*nqq. /noline mse cframe=ligr;
run;
```

The mysterious keyword `nqq.` is simply the normal quantiles. The three options in the plot statement are used to, respectively, delete the horizontal reference line, display mean squared error statistic on the graph and change the background of the graph from white to grey.

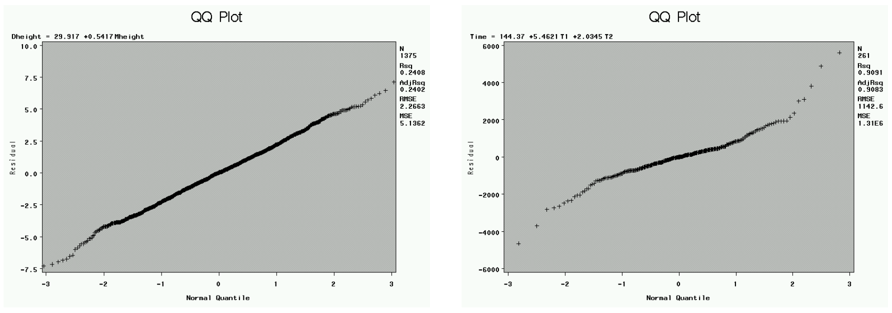


Fig. 9.1 QQnorm plots of residuals for the **`heights`** data (left) and for the **`transact`** data (right).

10 Variable Selection

10.1 THE ACTIVE TERMS

The first example in this chapter uses randomly generated data. This can be helpful in trying to understand issues against a background where we know the right answers. Generating random data is possible in most statistical packages, though doing so may not be easy or intuitive.

SAS The function `rannor` is used in the `data` step to generate standard normal (psuedo) random numbers. `rannor` has an optional argument `seed` that can be any positive integer. By setting the seed, you will always use the exact same "random" numbers. (How can they be random if you always get the same ones?) If `seed` is set to -1, the system selects the starting values for the random numbers. Only one seed can be specified per data step. The data set generates data only one observation at a time, so you need a `do` loop to get a vector of random numbers. A `output` statement must be included in the loop, or else only the last generated value will be kept.

The following SAS program generates one hundred pseudo-random numbers for each x_i's and e, then y is generated as a function of x_1, x_2 and e. Finally, `proc reg` is called to fit a linear model of y on all the x_i's, as in ALR[10.1]. See the fitted model in Table 10.1.

```
data case1;  *create a dataset called case1;
  seed=20040622;  *use this seed to get the same random numbers;
  do i=1 to 100;  *repeat to get 100 observations;
    x1=rannor(seed);
    x2=rannor(seed);
    x3=rannor(seed);
    x4=rannor(seed);
    e=rannor(seed);
    y=1+x1+x2+e;
    output; *REQUIRED to keep what you just did;
  end; *all do loops end with 'end';
* the next line prints the first 3 and last 3 observations;
proc print data=case1; where (i le 3 | i ge 98); run;
proc reg data=case1;
  model y=x1 x2 x3 x4/vif;
run;
```

Table 10.1 Excerpt of `proc reg` output with simulated independent predictors.

```
                 Parameter Estimates
               Parameter  Standard                      Variance
Variable   DF   Estimate     Error  t Value  Pr > |t|  Inflation
Intercept   1    0.99662   0.10738     9.28    <.0001          0
x1          1    0.97910   0.10870     9.01    <.0001    1.01447
x2          1    0.85222   0.10990     7.75    <.0001    1.01517
x3          1    0.01066   0.10833     0.10    0.9218    1.00133
x4          1   -0.06222   0.11079    -0.56    0.5757    1.01860
```

The second case in ALR[9.1] involves a non-diagonal variance-covariance matrix, Σ. Most computer programs, including SAS, can only generate uncorrelated random numbers. Suppose that X is an $n \times p$ matrix, all of whose elements are $\mathrm{N}(0, 1)$, random numbers, and suppose further we can find a matrix C such that $\Sigma = C'C$. This matrix C is called a *square root* of Σ. Then the matrix $X^* = XC'$ will be a matrix of random numbers, still with mean zero, but the covariance matrix will now be $C'C = \Sigma$.

The square root of a matrix is not unique, but any square root will work. `proc iml` has several routines for computing a square root; we will use the one called the *Cholesky decomposition* computed by the `root` macro.

```
data temp (drop=y); *set case1; *temp = case1 with y dropped;
proc iml;           *use proc iml to do computing
use temp;
read all var {x1 x2 x3 x4} into x; *x is a matrix;
read all var {e} into e;
SIGMA={  1     0  .95     0,
         0     1    0  -.95,
       .95     0    1     0,
         0  -.95    0     1}; *define SIGMA;
r=root(SIGMA); *compute its Cholesky factorization
x=x*r; *each row of this x has a 4-dim;
       *multivariate normal distribution;
       *with mean 0 and variance SIGMA;
x1=x[,1]; x2=x[,2]; x3=x[,3]; x4=x[,4]; *convert to vectors;
```

```
create case2 var {x1 x2 x3 x4 e};       *create new data set;
append var {x1 x2 x3 x4 e};
close case2;   *close the data set;
quit;          *quit iml;
data case2; set case2; y=x1+x2+e;
proc reg data=case2;
  model y=x1-x4/vif;
run;
```

The response y is generated from the same function of x_i's and e, and we fit the same linear model of y on x_i's, as in Case 1. See output in Table 10.2. Unlike the results in the book, in this particular random data set the active predictors are correctly identified, as will sometimes happen. As correlations get bigger, the frequency of correct identification gets lower.

Table 10.2 `proc reg` with simulated correlated predictors.

```
                Parameter Estimates
               Parameter  Standard                          Variance
Variable   DF   Estimate     Error  t Value  Pr > |t|  Inflation
Intercept   1   -0.00338   0.10738    -0.03    0.9749          0
X1          1    0.94667   0.34410     2.75    0.0071   10.16563
X2          1    0.66293   0.36497     1.82    0.0725   11.19670
X3          1    0.03413   0.34695     0.10    0.9218   10.15189
X4          1   -0.19926   0.35480    -0.56    0.5757   11.16227
```

10.1.1 Collinearity

SAS Variance inflation factors are computed in `proc reg` by adding the `/ vif` option to the `model` statement. In the two simulations, the *VIF* are all close to one for the first simulation, and equal to about 11 in the second; see output in Tables 10.1-10.2.

```
proc reg data=case1;
  model y=x1 x2 x3 x4 /vif;
run;
proc reg data=case2;
  model y=x1 x2 x3 x4 /vif;
run;
```

10.1.2 Collinearity and variances

10.2 VARIABLE SELECTION

10.2.1 Information criteria

The information criteria ALR[E10.7]–ALR[E10.9] depend only on the *RSS*, p', and possibly an estimate of σ^2, and so if these are needed for a particular model, they can be computed from the usual summaries available in a fitted regression model.

SAS We can retrieve *AIC*, *BIC*, C_p and *PRESS* statistics directly from `proc reg`. For `proc reg` in SAS, the definitions of *AIC*, C_p and *PRESS* are the same as ALR[E10.7], ALR[E10.9] and ALR[E10.10]. However, the definition of *BIC* is different from ALR[E10.8]. It is defined in SAS as

$$BIC = n\log(RSS_C/n) + 2(p+2)q - 2q^2$$

where

$$q = n\hat{\sigma}^2/RSS_C$$

BIC in ALR[E10.8] is called *SBC* in SAS. Since the `fullmodel` and the `submodel` in the following program produce different estimates of the error variance σ^2, each model uses its own $\hat{\sigma}$ in the calculation of C_p. This means that the C_p statistic from the `submodel` is *not* the value described in ALR. See output in Table 10.3.

```
data highway;
  set alr3.highway;
  logLen=log2(Len); logADT=log2(ADT);
  logTrks=log2(Trks); logSigs1=log2((Len*Sigs+1)/Len);
*create dummies for variable Hwy;
  if (Hwy eq 1) then Hwy1=1; else Hwy1=0;
  if (Hwy eq 2) then Hwy2=1; else Hwy2=0;
  if (Hwy eq 3) then Hwy3=1; else Hwy3=0;
  logRate=log2(Rate);
proc reg data=highway outest=m1
  (keep=_model_ _aic_ _bic_ _cp_ _press_ _p_ _sse_ _mse_ _sbc_);
  fullmodel: model logRate=logADT logTrks Lane Acpt logSigs1 Itg Slim
              logLen Lwid Shld Hwy1 Hwy2 Hwy3
              /noprint aic bic cp press sse mse sbc;
  submodel: model logRate=logLen Slim Acpt logTrks Shld
              /noprint aic bic cp press sse mse sbc;
run;
goptions reset=all;
title 'Model fit and diagnostic statistics for the highway data';
proc print data=m1; run;
*print some statistics for the full model, as;
*in alr{10.2.1}. BIC as in alr{E10.7} is displayed as SBC;
```

Table 10.3 Combined output of information criteria with the `highway` data.

```
       Model fit and diagnostic statistics for the highway data

Obs  _MODEL_    _PRESS_ _P_  _SSE_    _MSE_    _CP_   _AIC_     _BIC_     _SBC_
 1  fullmodel  11.2722  14 3.53696 0.14148   14  -65.6115 -48.5587 -42.3216
 2  submodel    7.6880   6 5.01595 0.15200    6  -67.9866 -63.8709 -58.0052
```

10.2.2 Computationally intensive criteria

Computation of *PRESS*, ALR[E10.10], is not common in regression programs, but it is easy to obtain given the residuals and leverages from a fitted model.

SAS SAS does allow computing *PRESS* by adding `/ press` to the model statement in `proc reg`. The results are shown in Table 10.3 using the program in Section 10.2.1.

10.2.3 Using subject-matter knowledge

10.3 COMPUTATIONAL METHODS

SAS The SAS procedure `proc reg` has an option `selection` that is added to the `model` statement for choosing a numerical procedure for selecting a subset of terms. While several choices are available for this option, the most relevant seem to be `forward` for forward selection; `backward` for backward elimination, and `cp`, to find the subsets that minimize C_p. This latter option users the Furnival and Wilson (1974) algorithm discussed in ALR[10.3]. Since `proc reg` does not permit factors, using C_p is essentially the same as using any of the other criteria discussed in ALR. If you have factors, you must code them yourself using dummy variables (an example is below), and you apparently can't force terms into every mean function, and add/delete terms as a group.

The first example uses the Furnival and Wilson algorithm to find subsets that minimize Mallow's C_p. As in ALR[10.3], we use the `highway` data. We start by coding the factor *Hwy* as a set of dummy variables.

```
goptions reset=all;
data highway;
  set alr3.highway;
  logLen=log2(Len); logADT=log2(ADT);
  logTrks=log2(Trks); logSigs1=log2((Len*Sigs+1)/Len);
  logRate=log2(Rate);
  if (Hwy eq 1) then Hwy1=1; else Hwy1=0;
  if (Hwy eq 2) then Hwy2=1; else Hwy2=0;
  if (Hwy eq 3) then Hwy3=1; else Hwy3=0;
proc reg data=highway;
  model logRate=logADT logTrks Lane Acpt logSigs1 Itg Slim
```

```
              logLen Lwid Shld Hwy1 Hwy2 Hwy3
              /selection=cp aic best=10;
run; quit;
```

We have requested the 10 subsets with minimum C_p, and have also requested that AIC be printed for each of these subsets. The output is shown in Table 10.4. Many of these subsets include one, but not all, of the dummy variables for Hwy, so these are not likely to be of much interest.

Table 10.4 Variable selection based on Mallow's C_p for the `highway` data.

```
C(p) Selection Method

Number in
  Model       C(p)  R-Square        AIC  Variables in Model

      7     3.4857    0.7789   -75.3600  logADT logTrks Acpt
                                         logSigs1 Slim logLen Hwy1
      5     3.5203    0.7453   -73.8303  logSigs1 Slim logLen Lwid Hwy2
      5     3.5544    0.7450   -73.7868  logADT logSigs1 Slim logLen Hwy1
      7     3.5744    0.7782   -75.2297  logADT Acpt logSigs1 Slim
                                         logLen Hwy1 Hwy2
      5     3.6065    0.7445   -73.7204  logSigs1 Slim logLen Shld Hwy2
      6     3.6289    0.7611   -74.3254  logADT logTrks logSigs1
                                         Slim logLen Hwy1
      6     3.7028    0.7604   -74.2249  logSigs1 Itg Slim logLen
                                         Hwy2 Hwy3
      7     3.8174    0.7762   -74.8747  logADT logSigs1 Slim logLen
                                         Lwid Hwy2 Hwy3
      7     3.9278    0.7753   -74.7143  logADT logSigs1 Slim logLen
                                         Hwy1 Hwy2 Hwy3
      7     3.9477    0.7751   -74.6855  logADT logSigs1 Slim logLen
                                         Lwid Hwy1 Hwy2
```

The program below illustrates forward selection.

```
proc reg data=highway;
  model logRate=logADT logTrks Lane Acpt logSigs1 Itg Slim
              logLen Lwid Shld Hwy1 Hwy2 Hwy3
              /selection=forward;
  ods output SelectionSummary=submodels;
run; quit;
proc print data=submodels; run;
```

The summary of forward selection from `proc reg` is given in Table 10.5.

Table 10.5 Summary of forward variable selection for the `highway` data.

```
Summary of Forward Selection

                           Var      Number Partial   Model
Obs Model   Dependent Step Entered    In    RSquare  Rsquare        Cp FValue   ProbF
 1  MODEL1   logRate    1  Slim        1    0.4765   0.4765   27.7232  33.68  <.0001
 2  MODEL1   logRate    2  logLen      2    0.1629   0.6394   10.2021  16.27  0.0003
 3  MODEL1   logRate    3  Acpt        3    0.0354   0.6748    7.9587   3.81  0.0589
 4  MODEL1   logRate    4  logTrks     4    0.0212   0.6961    7.4145   2.38  0.1325
 5  MODEL1   logRate    5  Hwy2        5    0.0192   0.7152    7.1199   2.22  0.1458
 6  MODEL1   logRate    6  logSigs1    6    0.0386   0.7539    4.4903   5.02  0.0321
 7  MODEL1   logRate    7  logADT      7    0.0158   0.7697    4.5933   2.13  0.1544
 8  MODEL1   logRate    8  Hwy3        8    0.0171   0.7868    4.5427   2.41  0.1312
```

10.3.1 Subset selection overstates significance

10.4 WINDMILLS

10.4.1 Six mean functions

10.4.2 A computationally intensive approach

The data for the windmill example in ALR[10.4.2] is not included with the `alr3` library, and must be downloaded separately from `www.stat.umn.edu/alr`.

SAS The SAS code used to compute ALR[F10.1] are given in the script for this chapter.

11 Nonlinear Regression

11.1 ESTIMATION FOR NONLINEAR MEAN FUNCTIONS

11.2 INFERENCE ASSUMING LARGE SAMPLES

SAS SAS includes several procedures and tools for fitting nonlinear regression models. We will discuss only two of them, `proc nlin` and `proc nlmixed`. `proc nlmixed` is newer and more general, and so we will mainly discuss using this procedure. An example of using `proc nlin` is also given. The syntax in these two procedures is much more complicated than is the syntax for `proc reg` or `proc glm` because these two programs are more general.

We begin by illustrating fitting a mean function

$$\mathrm{E}(Gain|A) = \theta_1 + \theta_2(1 - e^{-\theta_3 A}) \tag{11.1}$$

with the `turk0` data.

Here are the commands needed for `proc nlmixed`:

```
proc nlmixed data=alr3.turk0 corr;
  parms th1=620 th2=200 th3=10 sig2=10;
  fitGain=th1+th2*(1-exp(-th3*A));
  model Gain~Normal(fitGain,sig2);
  predict fitGain out=ghat;
run; quit;
```
Plot the fitted function
```
goptions reset=all;
symbol1 v=circle c=black;
```

```
symbol2 v=point c=black i=join l=1;
proc gplot data=ghat;
  plot Gain*A=1 Pred*A=2 /overlay hminor=0 vminor=0;
run; quit;
```

Table 11.1 Modified output from `proc nlmixed` for the `turk0` data.

```
The NLMIXED Procedure

                        Iteration History
Iter     Calls    NegLogLike        Diff     MaxGrad       Slope

   1         3    733.498721    1022.171     238.637    -2272.58
   2         4    658.723273    74.77545    120.8772    -502.002
... ...
  24        46    152.343619    0.000015    0.000243    -0.00003
  25        48    152.343619    2.268E-8    0.000019    -4.24E-8
         NOTE: GCONV convergence criterion satisfied.

             Fit Statistics

-2 Log Likelihood                  304.7
AIC (smaller is better)            312.7
AICC (smaller is better)           314.0
BIC (smaller is better)            318.9

                        Parameter Estimates
                       Standard
Parameter   Estimate      Error     DF   t Value   Pr > |t|    Alpha

th1           622.96     5.6064     35    111.12     <.0001     0.05
th2           178.25    10.9596     35     16.26     <.0001     0.05
th3           7.1222     1.1071     35      6.43     <.0001     0.05
sig2          353.35    84.4669     35      4.18     0.0002     0.05

            Parameter Estimates
Parameter      Lower       Upper    Gradient

th1           611.58      634.34    5.031E-6
th2           156.00      200.50    1.557E-6
th3           4.8748      9.3697    0.000019
sig2          181.88      524.83     -3.1E-7

            Correlation Matrix of Parameter Estimates
 Row    Parameter          th1          th2          th3         sig2
   1    th1             1.0000      -0.3388      -0.3928     0.000012
   2    th2            -0.3388       1.0000      -0.6052     -3.17E-6
   3    th3            -0.3928      -0.6052       1.0000     -4.21E-6
   4    sig2          0.000012     -3.17E-6     -4.21E-6       1.0000
```

The `parms` statement specifies the parameters that will appear in the model. These include the θs, which we have called `th`, and also the variance σ^2 that we call `sig2`. Unlike `proc reg`, you must specify starting values for the parameter

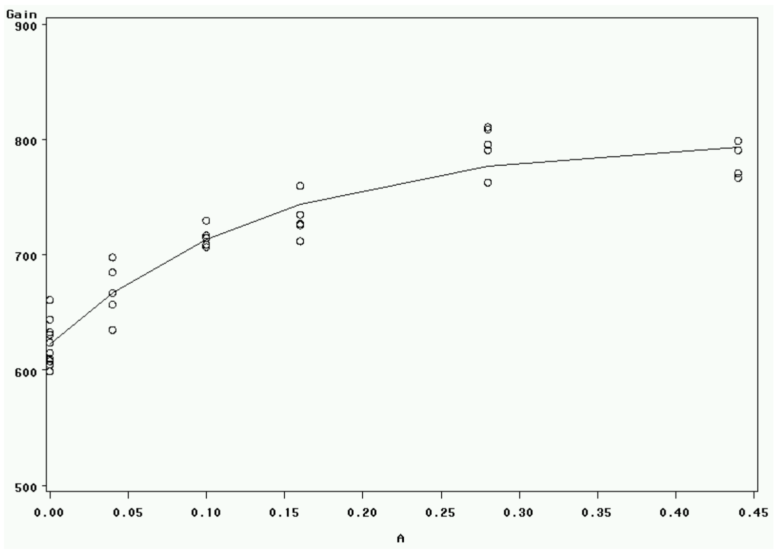


Fig. 11.1 Prediction curve of *Gain* on *A* for the `turk0` data.

estimates, as discussed in ALR[11.2], and if the starting values are poor, the computational algorithm may fail to find the estimates. Next, we define a mean function `fitGain` to be equal the kernel mean function, a rewriting of (11.1) using notation that SAS understands. The `model` statement specifies the *whole model, not just the mean function.* This corresponds to the use of the term "model" in ALR as referring to the mean function plus any other assumptions. For the example, the model assumes that *Gain* is normally distributed with mean `fitGain` and constant variance `sig2`. The option `corr` in the `proc nlmixed` statement tells SAS to display the correlation matrix of the parameter estimates. `proc nlmixed` permits many other distributions besides the normal. In addition, it has another statement called `random` for specifying random effects that we will not be using in this primer, except for computing the random coefficient models presented in Section 6.5. Finally, the `predict` statement is used to get a prediction and prediction standard error for each observation. The option `out` saves the output as a data file that is required to draw the graph of the fitted function. The output in Table 11.1 includes the parameter estimates and the estimated correlation matrix. The remaining part of the program uses `proc gplot` to graph the fitted curve and the data shown in Figure 11.1.

The next SAS program will give a lack-of-fit test, as shown in Table 11.2, for the above nonlinear model of *Gain* on *A*, using the `turk0` data:

```
goptions reset=all;
data turk0;
  infile "c:/My Documents/alr3/turk0.txt" firstobs=2;
  input A Gain;
proc nlmixed data=turk0;
  parms th1=620 th2=200 th3=10 s2=10;
  fitGain=th1+th2*(1-exp(-th3*A));
  model Gain~Normal(fitGain,s2);
  ods output ParameterEstimates=a1;
run; quit;
data a1; set a1; where (Parameter='sig2'); SS=Estimate*DF; DF=32;
proc glm data=turk0;
  class A;
  model Gain=A;
  ods output OverallAnova=a2;
run; quit;
data a2; set a2; where (Source='Error');
data lof (keep=DF SS); set a1 a2;
proc sort data=lof; by descending DF;
proc iml;
use lof;
read all var {DF} into resDF;
read all var {SS} into resSS;
DF=j(2,1,.); Sumsq=j(2,1,.); F=j(2,1,.); pval=j(2,1,.);
DF[2]=resDF[1]-resDF[2];
Sumsq[2]=resSS[1]-resSS[2];
F[2]=Sumsq[2]/DF[2]/(resSS[2]/resDF[2]);
pval[2]=1-probF(F[2],DF[2],resDF[2]);
title 'lack of fit test';
print resDF resSS DF Sumsq F pval;
quit;
```

`proc iml` is used to do the F-test comparing the nonlinear model to the simple one-way ANOVA, since `proc nlmixed` has neither `class` statement nor `test` statement.

Table 11.2 Lack of fit test for a nonlinear model with the `turk0` data.

```
                  lack of fit test

   RESDF     RESSS        DF     SUMSQ         F       PVAL

      32  12367.34         .         .         .          .
      29    9823.6         3 2543.7399 2.5031033 0.0789298
```

For fitting nonlinear models with factors, you need to define dummy variables for factor variables in the data step. Then you can fit nonlinear model with these dummies. We fit four different models and compare them with F-tests. Since `proc nlmixed` does not provide correct error degrees of freedom, we present such an example using `proc nlin` for the `turkey` data. The syntax for `nlin` is a little simpler for nonlinear regression models because the method is much less general. The `parms` statement is the same as in `proc nlmixed`.

The `model` statement in `nlin` specifies only the mean function, as constant variance and normality as assumed by this procedure.

```
goptions reset=all;
data turkey;
  set alr3.turkey;
  S1=(S eq 1);
  S2=(S eq 2);
  S3=(S eq 3);
  w=sqrt(m);
  wGain=w*Gain;
proc nlin data=turkey; *most general;
  parms th11=620 th12=620 th13=620
th21=200 th22=200 th23=200
th31=10 th32=10 th33=10;
  model wGain=w*(S1*(th11+th21*(1-exp(-th31*A)))+
S2*(th12+th22*(1-exp(-th32*A)))+
S3*(th13+th23*(1-exp(-th33*A))));
  ods output Anova=a1;
run;
proc nlin data=turkey; *common intercept;
  parms th1=620
th21=200 th22=200 th23=200
th31=10 th32=10 th33=10;
  model wGain=w*(th1+
S1*(th21*(1-exp(-th31*A)))+
S2*(th22*(1-exp(-th32*A)))+
S3*(th23*(1-exp(-th33*A))));
  ods output Anova=a2;
run;
proc nlin data=turkey; *common intercept and asymptote;
  parms th1=620 th2=200
th31=10 th32=10 th33=10;
  model wGain=w*(th1+th2*(
S1*(1-exp(-th31*A))+
S2*(1-exp(-th32*A))+
S3*(1-exp(-th33*A))));
  ods output Anova=a3;
run;
proc nlin data=turkey; *common regression;
  parms th1=620 th2=200 th3=10;
  model wGain=w*(th1+th2*(1-exp(-th3*A)));
  ods output Anova=a4;
run;
*************************************;
***Anova table for model 1, 2 and 4***;
*************************************;
data anova (keep=DF SS); set a1 a2 a4; where (Source eq 'Residual');
proc sort data=anova; by descending DF;
proc iml;
use anova;
read all var {DF} into resDF;
read all var {SS} into resSS;
DF=j(3,1,.); Sumsq=j(3,1,.); F=j(3,1,.); pval=j(3,1,.);
DF[2]=resDF[1]-resDF[2]; DF[3]=resDF[2]-resDF[3];
Sumsq[2]=resSS[1]-resSS[2]; Sumsq[3]=resSS[2]-resSS[3];
F[2]=Sumsq[2]/DF[2]/(resSS[2]/resDF[2]);
F[3]=Sumsq[3]/DF[3]/(resSS[3]/resDF[3]);
pval[2]=1-probF(F[2],DF[2],resDF[2]);
pval[3]=1-probF(F[3],DF[3],resDF[3]);
title 'Analysis of Variance Table';
print resDF resSS DF Sumsq F pval;
quit;
*************************************;
***Anova table for model 1, 3 and 4***;
*************************************;
data anova (keep=DF SS); set a1 a3 a4; where (Source eq 'Residual');
proc sort data=anova; by descending DF;
proc iml;
use anova;
read all var {DF} into resDF;
read all var {SS} into resSS;
DF=j(3,1,.); Sumsq=j(3,1,.); F=j(3,1,.); pval=j(3,1,.);
DF[2]=resDF[1]-resDF[2]; DF[3]=resDF[2]-resDF[3];
Sumsq[2]=resSS[1]-resSS[2]; Sumsq[3]=resSS[2]-resSS[3];
F[2]=Sumsq[2]/DF[2]/(resSS[2]/resDF[2]);
F[3]=Sumsq[3]/DF[3]/(resSS[3]/resDF[3]);
pval[2]=1-probF(F[2],DF[2],resDF[2]);
pval[3]=1-probF(F[3],DF[3],resDF[3]);
title 'Analysis of Variance Table';
print resDF resSS DF Sumsq F pval;
quit;
```

We fit weighted nonlinear regression models with one continuous variable and one factor. Model 1, 2, 3 and 4 are *most general*, *common intercept*, *common intercept and asymptote* and *common regression*, respectively. We first code the dummies *S1*, *S2* and *S3* for *S*, and create the weighted response in the data step, namely, *wGain* here, using $\sqrt{m}$ as weights. Then we fit four different nonlinear models and run two ANOVA for comparing model 1, 2, 4 and comparing models 1, 3, 4, respectively. See ANOVA tables in Table 11.3. Finally, we compute all the F-values and p-values in `proc iml`.

Table 11.3 ANOVA tables for comparing four different nonlinear models with the `turk0` data.

```
***ANOVA table for comparing model 1, 2 and 4***;
             Analysis of Variance Table

RESDF    RESSS    DF    SUMSQ         F     PVAL
    10 4326.0797     .         .         .         .
     6 2040.0059     4 2286.0738 1.6809318 0.2710972
     4 1151.1523     2 888.85359 1.5442849 0.3184218

***ANOVA table for comparing model 1, 3 and 4***;
             Analysis of Variance Table

RESDF    RESSS    DF    SUMSQ         F     PVAL
    10 4326.0797     .         .         .         .
     8 2568.3882     2 1757.6915 2.7374233 0.1242409
     4 1151.1523     4 1417.2359 1.2311454 0.4225784
```

11.3 BOOTSTRAP INFERENCE

SAS We give code for the bootstrap discussed in ALR[11.3]. See also the documentation for the macro `boots` for more information. The result with B=999 and `seed`=2004 is given in Table 11.4.

```
******************************************;
***A similar macro appeared in chapter 4***;
******************************************;
%let n=39; *#obs in the data;
%macro nlsboot(B=1, seed=1234, outputdata=temp);
%do i=1 %to &B;
*the following command tells SAS to clear both;
*the log window and the output window;
dm 'log; clear; output; clear; ';
data analysis;
  choice=int(ranuni(&seed*&i)*&n)+1;
  set alr3.segreg point=choice;
  j+1;
  if j>&n then STOP;
proc nlmixed data=analysis;
  parms th0=70 th1=.5 gamma=40 sig2=1;
  fitC=th0+th1*max(0,Temp-gamma);
  model C~Normal(fitC,sig2);
  ods output ParameterEstimates=pe;
run; quit;
proc sql; create table outests as (select Estimate,
  sum(Estimate*(Parameter='th0')) as th0,
  sum(Estimate*(Parameter='th1')) as th1,
  sum(Estimate*(Parameter='gamma')) as gamma from pe);
quit;
```

Table 11.4 Summary of bootstrap estimation for the `segreg` data with `proc nlmixed`, based on 999 resamples.

```
              th0        th1      gamma
Mean     74.757751  0.6059323  42.874274
SD       1.6226234  0.1274196  5.1164431
2.5%     71.472186  0.4577865         36
97.5%    77.657143  0.9673802  54.599069
```

```
data outests (keep=th0 th1 gamma); set outests (obs=1);
proc append base=&outputdata data=outests; run;
%end;
%mend nlsboot;
***example as in the book, with B=999 resample estimates***;
%nlsboot(B=999, seed=2004, outputdata=mynlboot);
proc univariate data=mynlboot noprint;
  var th0 th1 gamma;
  output out=mynlbootstat mean=mean_th0 mean_th1 mean_gamma
         std=sd_th0 sd_th1 sd_gamma
         pctlpts=2.5, 97.5 pctlpre=th0 th1 gamma;
run;
goptions reset=all;
proc iml; *we launch proc iml to have a formatted output;
use mynlbootstat;
read all var _all_ into summary;
out=j(4,3,.);
out[1,]=t(summary[1:3]);
out[2,]=t(summary[4:6]);
out[3,]=t(summary[{7,9,11}]);
out[4,]=t(summary[{8,10,12}]);
print out[rowname={'Mean' 'SD' '2.5%' '97.5%'}
          colname={'th0' 'th1' 'gamma'} label=""];
quit;
```

We obtain bootstrap estimate by taking average over all available estimates from the resamples, bootstrap standard error for each estimate is given by the sample standard deviation of B=999 sets of estimates available. Accompanied are 95% confidence intervals.

11.4 REFERENCES

12 Logistic Regression

Both logistic regression and the normal linear models that we have discussed in earlier chapters are examples of *generalized linear models.* Many programs, including SAS, R, and S-Plus, have procedures that can be applied to any generalized linear model. Both JMP and SPSS seem to have separate procedures for logistic regression. There is a possible source of confusion in the name. Both SPSS and SAS use the name *general linear model* to indicate a relatively complex linear model, possibly with continuous terms, covariates, interactions, and possibly even random effects, but with normal errors. Thus the general linear model is a special case of the generalized linear models.

12.1 BINOMIAL REGRESSION

12.1.1 Mean Functions for Binomial Regression

12.2 FITTING LOGISTIC REGRESSION

SAS There is more than one way to fit logistic regression models in SAS, using, for example, `proc logistic` and `proc genmod`. We choose to use `proc genmod` that not only fits binary regression models, but also other generalized linear models such as Poisson and gamma regression (and linear models, too). This procedure can be used to fit logistic model with the option `/dist=binomial link=logit`. When SAS fits a logistic regression model with a response y equal to 0 or 1, and terms x, it will by default model $\text{Prob}(y = 0|x)$, which is not standard. You can get SAS to model Prob($y = 1|x$) by adding the option `descending` after the `proc genmod` statement; see examples in the next section.

12.2.1 One-predictor example

SAS First we fit logistic regression with one term. In this example, the variable y contains only two different levels, 0 and 1. The option `descending` in the `proc genmod` statement tells SAS to model Prob($y = 1|x$) instead of Prob($y = 0|x$).

```
data blowBF;
  set alr3.blowBF;
  logD=log2(D);
proc genmod descending data=blowBF;
  model y=logD /dist=binomial link=logit;
run; quit;
```

Edited output is given in Table 12.1. All the values shown are described in ALR, except the "Wald 95% confidence limits", which are simply given by the estimate plus or minus 1.96 times the standard error of the estimate.

Table 12.1 Output from `proc genmod` for the `blowBF` data.

```
             Criteria For Assessing Goodness Of Fit
     Criterion                 DF           Value        Value/DF
     Deviance                 657        655.2420          0.9973
     Scaled Deviance          657        655.2420          0.9973
     Pearson Chi-Square       657        677.4535          1.0311
     Scaled Pearson X2        657        677.4535          1.0311
     Log Likelihood                     -327.6210

                Analysis Of Parameter Estimates
                        Standard Wald 95% Confidence    Chi-
Parameter  DF  Estimate   Error        Limits        Square Pr > ChiSq
Intercept   1   -7.8925   0.6326  -9.1322   -6.6527   155.68     <.0001
logD        1    2.2626   0.1914   1.8875    2.6378   139.74     <.0001
Scale       0    1.0000   0.0000   1.0000    1.0000
```

If the response is the number of events out of a known number of trials, the model statement will look like `model y/m=x1 x2;`, where y is the variable name for the number of events and m is the variable name for the number of trials. We will illustrate this in the next section with the `titanic` data.

ALR[F12.1–3] are provided more as pedagogical tools than as data analytic tools, and in any case they are difficult to draw using SAS. We provide programs for them in the scripts for this chapter.

12.2.2 Many Terms

SAS Like `proc glm`, `proc genmod` permits the use of factors and interactions in model statements. In the example below, a `class` statement is added to tell SAS to treat *Class*, *Age* and *Sex* as factors. We also illustrate the use of `contrast` statements make comparisons between levels of a factor or between terms[1]. Choose the name for the contrast, in quotation marks. Then if you want to test different levels of a factor, give its name followed by contrast coefficients; the number of coefficients should equal the number of levels of the factor. If you want to test the parameters for two continuous variables, say $\beta_{x_1} = \beta_{x_2}$, you would have a contrast statement like: `contrast "x1?=x2" x1 1 x2 -1 /wald;`. The `wald` option tells SAS to use the Wald chi-square statistic, which presumably is computed by squaring the estimated contrast divided by its standard error. The following binomial model uses the `titanic` data, and the output is given in Table 12.2.

```
goptions reset=all;
proc genmod data=alr3.titanic;
  class Class Age Sex;
  model Surv/N=Class Age Sex / dist=binomial link=logit;
  contrast "Class crew?=Class first" Class 1 -1 0 0 /wald;
  contrast "Class second?=Class third" Class 0 0 1 -1 /wald;
  contrast "Female?=Male" Sex 1 -1 /wald;
run; quit;
```

The default parameterization method for factors is the same as used in `proc glm`, which leaves off the last level. See Section 6.1.3 for details.

In `proc genmod`, you can define interactions and higher order polynomial terms in the model statement. Here are two models listed in ALR[T12.2]. In the output, Wald chi-square tests are automatically done for each parameter. We use the `blowBF` data, and the output from the second `proc genmod` is given in Table 12.3.

```
goptions reset=all;
data blowBF;
  set alr3.blowBF;
  logD=log2(D);
proc genmod descending data=blowBF;
  model y=logD S /dist=binomial link=logit;
  output out=many1 p=pred_prob;
run; quit;
proc genmod descending data=blowBF;
  model y=logD S logD*S /dist=binomial link=logit;;
  output out=many2 p=pred_prob;
run; quit;
```

Programs for jittered scatterplots and for ALR[F12.5] are given in the script for this chapter.

[1]This statement can also be used with `proc glm`.

Table 12.2 Edited output from a binomial model using `proc genmod` for the `titanic` data.

```
The GENMOD Procedure
            Model Information

Data Set                      ALR3.TITANIC
Distribution                      Binomial
Link Function                        Logit
Response Variable (Events)            Surv
Response Variable (Trials)               N

Number of Observations Read          14
Number of Observations Used          14
Number of Events                    711
Number of Trials                   2201

           Criteria For Assessing Goodness Of Fit

Criterion                DF           Value        Value/DF

Deviance                  8        112.5666         14.0708
Scaled Deviance           8        112.5666         14.0708
Pearson Chi-Square        8        103.8296         12.9787
Scaled Pearson X2         8        103.8296         12.9787
Log Likelihood                   -1105.0306

                             Analysis Of Parameter Estimates

                               Standard   Wald 95% Confidence
Parameter         DF Estimate    Error        Limits      Square  P-value

Intercept          1  -1.0924   0.2370  -1.5570  -0.6279   21.24  <.0001
Class      Crew    1   0.9201   0.1486   0.6289   1.2113   38.34  <.0001
Class      First   1   1.7778   0.1716   1.4415   2.1140  107.37  <.0001
Class      Second  1   0.7597   0.1764   0.4140   1.1053   18.56  <.0001
Class      Third   0   0.0000   0.0000   0.0000   0.0000     .      .
Age        Adult   1  -1.0615   0.2440  -1.5398  -0.5833   18.92  <.0001
Age        Child   0   0.0000   0.0000   0.0000   0.0000     .      .
Sex        Female  1   2.4201   0.1404   2.1449   2.6953  297.07  <.0001
Sex        Male    0   0.0000   0.0000   0.0000   0.0000     .      .
Scale              0   1.0000   0.0000   1.0000   1.0000

                            Contrast Results

                                          Chi-
Contrast                          DF     Square    Pr > ChiSq    Type

Class crew?=Class first            1      29.71        <.0001    Wald
Class second?=Class third          1      18.56        <.0001    Wald
Female?=Male                       1     297.07        <.0001    Wald
```

Table 12.3 Output from `proc genmod` with interaction for the `blowBF` data.

```
             Criteria For Assessing Goodness Of Fit
    Criterion                DF          Value       Value/DF
    Deviance                655       541.7456         0.8271
    Scaled Deviance         655       541.7456         0.8271
    Pearson Chi-Square      655       885.6535         1.3521
    Scaled Pearson X2       655       885.6535         1.3521
    Log Likelihood                   -270.8728

Algorithm converged.

                 Analysis Of Parameter Estimates
                        Standard  Wald 95% Confidence   Chi-
Parameter  DF  Estimate    Error         Limits       Square Pr > ChiSq
Intercept   1   -3.6782   1.4252   -6.4716   -0.8848    6.66     0.0099
logD        1    0.4007   0.4390   -0.4597    1.2611    0.83     0.3614
S           1  -11.2054   3.6387  -18.3371   -4.0738    9.48     0.0021
logD*S      1    4.9108   1.1403    2.6758    7.1457   18.55     <.0001
Scale       0    1.0000   0.0000    1.0000    1.0000
```

12.2.3 Deviance

SAS In the following example, we retrieve the `Deviance` and the error `DF` from the output of `proc genmod`. We then store these information in the data set `test`, and we use `proc iml` to do all Chi-square tests. Output is formatted so that *ANOVA* results are presented in Table 12.4.

```
goptions reset=all;
data blowBF;
  set alr3.blowBF
  logD = log(D);
proc genmod descending data=blowBF;
  model y=logD / dist=binomial link=logit;
  ods output ModelFit=m1;
run; quit;
proc genmod descending data=blowBF;
  model y=logD S / dist=binomial link=logit;
  ods output ModelFit=m2;
run; quit;
proc genmod descending data=blowBF;
  model y=logD S logD*S/ dist=binomial link=logit;
  ods output ModelFit=m3;
run; quit;
data test (keep=DF value);
set m1 m2 m3; where (criterion eq 'Deviance');
proc iml;
use test;
read all var _all_ into x; *_all_ stands for all variables in data 'test';
resDF=x[,1]; resDev=x[,2];
```

```
Model={"m1", "m2", "m3"};
DF=j(3,1,.); Dev=j(3,1,.); pval=j(3,1,.);
DF[2]=resDF[1]-resDF[2]; DF[3]=resDF[2]-resDF[3];
Dev[2]=resDev[1]-resDev[2]; Dev[3]=resDev[2]-resDev[3];
pval[2]=1-probchi(Dev[2],DF[2]);
pval[3]=1-probchi(Dev[3],DF[3]);
title 'Analysis of Variance Table for model m1, m2 and m3';
print Model resDF resDev DF Dev pval;
quit;
```

Table 12.4 ANOVA for comparing 3 logistic models using `proc iml` for the `blowBF` data.

```
   Analysis of Variance Table for model m1, m2 and m3

 MODEL   RESDF     RESDEV     DF        DEV       PVAL
 m1        657    655.242      .          .          .
 m2        656  563.90095      1  91.341046          0
 m3        655  541.74561      1  22.155343  2.5146E-6
```

Just to remind you that the keyword `_all_` that appears in the `proc iml` above simply tells SAS to read all variables in the referenced data set into this procedure.

12.2.4 Goodness of Fit Tests

SAS There are several ways to test lack-of-fit. One is the Deviance/ErrorDF type of statistics from `proc genmod`. It is treated as criterion for assessing goodness-of-fit. An example is given below, with output shown in Table 12.5:

```
goptions reset=all;
data blowBF;
  set alr3.blowBF;
  logD=log2(D);
proc genmod descending data=blowBF;
  model y=logD / dist=binomial link=logit;
run; quit;
```

Table 12.5 Criteria for assessing goodness of fit from `proc genmod` for the `blowBF` data.

```
             The GENMOD Procedure

    Criteria For Assessing Goodness Of Fit
Criterion              DF       Value   Value/DF
Deviance              657    655.2420     0.9973
Scaled Deviance       657    655.2420     0.9973
Pearson Chi-Square    657    677.4535     1.0311
Scaled Pearson X2     657    677.4535     1.0311
Log Likelihood              -327.6210
```

12.3 BINOMIAL RANDOM VARIABLES

12.3.1 Maximum likelihood estimation

12.3.2 The Log-likelihood for Logistic Regression

12.4 GENERALIZED LINEAR MODELS

Problems

SAS 12.5.1. SAS `proc insight`, or equivalently, the *Interactive Data Analysis* tool, can be used to assign colors to data points. After you launch the *Interactive Data Analysis* tool, choose Edit → Windows → Tools, which will open a graphical panel as a new window. You can hold and drag your mouse cursor to select data points in a rectangular region on any graph, then set color to the selected points by clicking on one of the color buttons in the "*SAStool*" window. SAS keeps track of the color, and any subsequent graphs will be in different colors. *Hint: The histogram of the response variable* Y *may help.*

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